OPTIMUM DESIGN OF ELASTIC CABLE STAYED STRUCTURES

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By

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TO MY PARENTS

JANAKI AND MAHADEVAN

FOR ALL THEIR EFFORTS

7/4/75

CERTIFICATE

This is to certify that the thesis entitled OPTIMUM DESIGN OF ELASTIC CABLE STAYED STRUCTURES by Mr. Mahadevan Hariharan for the award of the degree of Doctor of Philosophy is a record of bonafide research work carried out by him under my supervision, and has not been submitted elsewhere for a degree.

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LIST OF SYMBOLS

All symbols are explained when they appear first in the text. Only those symbols which appear more than once are listed here.

A – Area of cross section

A_c – Area of the cable

B – Width of flange of the box beam or I-section.

C_j – Cost of the j-th member,

$\vec{d}$ – Vector of member dimensions and pretension forces

D – Depth of the box beam or I-section

$FMAX_k$ – Normalising factor for the k-th equilibrium constraint

g_j – j-th inequality constraint

I – Moment of inertia of the cross section

K – Stiffness matrix of the structure

l_k – k-th equality constraint

M – Number of degrees of freedom of the structure

N – Number of design variables

NCC – Total number of inequality constraints.

NM – Number of members in the structure

$\vec{p}$ – Nodal force vector

P – Pretension force in a cable

Q – Axial force

r – Penalty parameter

S – Stiffness matrix of a member in element coordinate system

$\vec{u}$ - Nodal displacement vector

$\vec{x}$ - Design variable vector

Z - Section Modulus of the cross section

σ_{yc} - Yield stress or proof stress in the cable

Φ - Penalty function

SYNOPSIS

A computational capability for the automated optimum elastic design of general framed structures is developed in the present work. The optimum design of cable stayed structures, which are becoming quite popular as long span bridge girders is studied using the developed programs. Earlier studies in the analysis of this class of structures report that they exhibit geometrically nonlinear behaviour under working load due to the beam-column effect. The optimum design formulations are generalised to include this nonlinear behaviour.

The objective function chosen is the total cost of materi used, which is to be minimised subject to constraints on stresse and deformations in the structure. The bending elements of the class of structures studied are assumed to be box sections of specified width and depth. The thicknesses of the webs and flanges of these elements are the design variables. Constraints on the extreme fibre stress due to axial force and bending and on the average shear stress in the webs are considered. These elements are supported at discrete points by high tensile cables under an initial tension. The areas of the cables and the pretension forces are also considered as the design variable and constraints are imposed on the stresses in the cables due to initial tension and the stresses at working load.

The structural optimisation problem, when cast as a non-linear programming problem, can be formulated in two different ways: (1) Synthesis Approach and (2) Integrated Synthesis Approach. In the synthesis approach the geometric properties of the structure are considered as design variables, and constraints imposed on stresses and deformations. The structural analysis is performed explicitly to quantify the constraints by any of the well-known methods of structural analysis. The stiffness method of structural analysis is used in the present work. In the second approach, the equilibrium equations are appended as equality constraints, considering the set of displacements as additional design variables. This eliminates the solution of simultaneous equations everytime the objective function and constraints are evaluated. Such a formulation is possible for discrete structures like trusses and frames using the stiffness method of analysis and for continuous structures analysed using the finite element displacement method. The synthesis approach leads to a mathematical programming problem with nonlinear inequality constraints, and the integrated synthesis approach to a problem with both equality and inequality constraints. The objective function in both the formulations turns out to be a linear function of the design variables. The non-linear programming problem is solved using the Sequential Unconstrained Minimization Technique, using a quasi-Newton algorithm for unconstrained minimization.

The nonlinear beam-column effect in the frames is incorpo rated through the stiffness matrix of the elements. The secant stiffness matrix based on the conventional beam-column approach is used in the present work.

The computation of gradients required in the optimization procedure is generally performed by the finite difference technique. This procedure, however, involves a considerable amount of unnecessary and repetitive computations. Efficient procedure are developed and adopted for gradient computation in the presen work.

The two formulations mentioned above are applied to the minimum cost design of cable stayed structures in order to compa their efficiencies. The synthesis approach has been found to be quite efficient and reliable in the present study. The integrat synthesis approach, on the other hand, suffers from the disadvantage of proper scaling of the constraints to effect a smoot behaviour. Premature termination of the algorithm is experience for large problems and this can only be avoided by a proper choice of the scaling parameters.

The results of the example problems considered indicate that the minimum cost design exhibits the property of 'fully constrained design'. In other words, each geometrical design variable in the optimum design takes a value so that one of the stress constraints of the corresponding member is active or the

variable itself is at its lower bound. The optimum design, therefore, has at least as many bounded constraints as the number of geometric variables in the problem. The pretension forces in the cables alone are the free variables, and they govern the cost of the fully constrained design. Therefore, a third formulation is considered in the present work, in which the pretension forces in the cables are the only design variables, and at every design point, the fully constrained design is performed to find the cost. This formulation leads to an unconstrained minimization problem. The fully constrained design formulation is the most efficient in the sense that it is much faster than the synthesis approach.

The effect, on the optimum design, of variation of some of the input parameters, e.g. the ratio of cost of high tensile cable to that of mild steel, minimum thickness requirements, depth of the girder and the allowable stresses in the cable, are studied for both the linear elastic behaviour and the geometrically nonlinear behaviour and the results discussed.

Some of the salient conclusions are: (1) The optimum design of the structures considered is a fully constrained design in the sense defined earlier, and (2) for structures loaded only at the nodes, inclusion of geometrical nonlinearity reduces the minimum cost of the structure, while for structures loaded along the members, an increase in the optimum cost is experienced

The principal features of the present work are: (1) to compare the efficiencies of various optimal design formulations for cable stayed structures, and (2) to embed the geometrically nonlinear behaviour of framed structures in the optimum design framework and to study its effect on the optimum design of cable stayed structures which have been reported to exhibit this behaviour. The work reveals that this behaviour does not significantly influence the minimum cost design of this class of structures.

CHAPTER ONE

INTRODUCTION

1.1 GENERAL REMARKS:

Structural optimization may be defined as the rational evolution of a structural design which satisfies a set of functional requirements, and which, in terms of a predefined criterion, is the best of all such acceptable designs. Early attempts at the evolution of optimum designs led to the theorems of Maxwell |1| and Michell |2|, and to considerable amount of analytical work in the recent years. Theorems providing the best choice of configuration and disposition of materials for structural elements and systems have been stated and proved. The excellent surveys by Wasiutynski and Brandt |3| in 1963 and by Sheu and Prager |4| in 1968 provide a complete historical development of the analytical methods of structural optimization. The developments in the area in the past seven years warrant another critical review.

The advent of digital computers and their use in solving structural analysis and design problems led to the approach of Automated Optimum Structural Design. Developments in computer technology, providing the researchers with machines

of greater speed and storage capacity, have made this approach applicable to problems of varying degrees of complexity. Consequently, the methods of mathematical programming gained considerable impetus and their use to solve problems of different degrees of complexity has, by now, become widespread. The latest innovation with on-line computing systems is the interactive design in which the engineer is in constant communication with the machine during the design process. This development is yet in its infancy, and the current research in structural optimization is predominantly in the area of automated optimum design.

A large number of structural systems are idealised, analysed and designed as plane frames. Cable stayed structures form one such class of systems, which are quite popular among civil engineers. They are mainly used in bridges, in the form of cable stayed girders, and in communication systems, as guyed towers. The main structural members are supported at intervals by high tensile wires under some initial tension. The cables being axially loaded members, the structural material is most efficiently used in these elements, and hence the material consumption of these structures is quite low as compared with structures of other types.

The design of framed structures follows one of the following three design philosophies: a. Elastic Design, b. Plastic Design, and c. Limit State Design. Reinforced

concrete frames are designed by any one of the three philosophies, while steel structures are designed by either of the first two. The study of optimum design of framed structures has received considerable attention of the researchers in the past, as is evident from the enormous amount of literature available today |5,6|. Curiously enough, attention was primarily directed towards the plastic design of frames by linear programming, and the elastic and plastic design of simple frames and their components by variational methods and consequent optimality criteria. Relatively less attention has been paid to the elastic design of framed structures. The analytical (variational) approaches are useful in arriving at qualitative conclusions and optimality criteria for different cases. They, however, become extremely complicated and the solution seeking and interpretation of the optimality criteria become difficult for any but the simplest of structures. The efforts have therefore been restricted to the design of simple beams, columns and portals with continuously or discretely varying sections subjected to simple loads. The numerical methods, on the other hand, employ iterative or mathematical programming techniques, or a combination of both and they are used to design complex structures. However, qualitative conclusions for design cannot be directly obtained by the numerical methods. They are popular because they are the only means of solving practical problems of varied degrees of complexity. The analytical treatment of

simple structures, however, is necessary for the following reasons: It reveals faults in intuitive design criteria that have long been taken as self-evident (e.g. the simultaneous failure mode theory and the fully stressed design concept). Also, it provides a deeper insight into the optimality criteria, and enables the designer to choose a near-optimum design as a starting point for the optimization using the numerical techniques.

The plastic design problem, under certain simplifying assumptions, can be cast as a linear programming problem, which can be solved using standardised procedures. The major drawback of the general plastic design formulation is that the serviceability constraints under working load are not considered. The inclusion of these constraints would make the formulation a nonlinear programming problem. The elastic design invariably leads to a nonlinear programming problem which has no specific standardised solution procedure. Most of the effort in the elastic design has been directed towards developing efficient procedures to solve the resulting nonlinear programming problem. The complexities of nonlinear programming problem are probably the deterrants which divert the researchers away from elastic design, although it is one of the most challenging fields of optimal structural design.

The present work addresses itself to the automated optimum elastic design of cable stayed bridge girders. Three formulations leading to nonlinear programming problems have been studied and their relative efficiency discussed. The effects of inclusion of the geometrically nonlinear behaviour of the structure, and of the variation of some of the predetermined design parameters, on the optimum design, are studied.

1.2 THE PRESENT WORK:

The computational capability for the automated optimum design of planar framed steel structures under linear elastic behaviour is developed in the present work. The objective function chosen is the total cost of materials used, which is to be minimised subject to constraints on stresses and deformations in the structure. The design variables chosen are the cross sectional properties of the members.

The structural optimization problem, when cast as a nonlinear programming problem, can be formulated in two different ways: one in which the geometric properties of the structure are considered as design variables, and constraints imposed on stresses and deformations. The analysis is performed explicitly by any of the well-known methods of structural analysis. The stiffness method of analysis is adopted for this formulation in the present work. This formulation, henceforth, is termed the 'Synthesis Approach', the details of which are presented in

Section 2.2. Alternatively, the equilibrium equations are appende as equality constraints, considering the set of displacements as additional design variables. This eliminates the solution of simultaneous equations everytime the objective function and constraints are evaluated. Such a formulation is possible for discrete structures like trusses and frames using the stiffness method of analysis, and for continuous structures idealised and analysed using the finite element displacement method. This formulation, henceforth termed the 'Integrated Synthesis Approach', is described in Section 2.3. The synthesis approach leads to a mathematical programming problem with nonlinear inequality constraints and the integrated synthesis approach to a problem with both nonlinear equality and inequality constraints. The objective function in both the formulations turns out to be a linear function of the design variables.

The following are the general features of the computer programs developed for the two formulations mentioned above. They are independent of the structural configuration, in the sense that all the relevant data required for the optimization is automatically developed from the geometrical and other data of the structure, provided as input. The objective function, constraints and the gradients with respect to the design variables are evaluated implicitly at any design point. They also admit the possibility of different members being made of

different materials with different unit costs, and also of different shapes. Given the type of member, the program automatically chooses the design variables and the stress constraints applicable to that type of member. Such a capability has also been incorporated earlier in the works of Kavlie and Moe |7| for the optimum design of the component frames of ship structures.

The present work aims to study the optimum design of planar framed steel structures supported at discrete points by high tensile cables under initial tension, with particular reference to cable stayed bridge girders. The bending elements in these structures are subjected to a considerable amount of axial force and earlier works in the analysis of these structures report that they exhibit geometrically nonlinear behaviour due to the beam-column effect even under working loads. The above formulations are therefore generalised to incorporate this nonlinear effect with little modification to the computer programs developed. The relative merits of the two formulations are studied through the example problems.

The example problems solved during the course of this work show that the optimum design for this class of structures is a 'Fully Constrained Design'. In other words, each geometrical design variable in the optimum design takes a value so that one of the stress constraints of the corresponding member of

the structure is active or the variable itself is at its lower bound. The optimum design, therefore, has at least as many bounded constraints as the number of geometrical design variables in the problem. The pretension forces in the cables are the free variables, which control the behaviour of the structure. Therefore, a third formulation is considered in the present work, in which the pretension forces in the cables are the only design variables, and at every design point the 'fully constrained design' is performed. This formulation leads to an unconstrained minimization problem presented in Section 2.4. The computer program developed for this formulation also preserves the generality and the versatility of the earlier two.

General approaches to the analysis of geometrically nonlinear plane frames and the modifications required in the computer programs to incorporate the nonlinear effects are discussed in Chapter 3. The nonlinearity in the present work is included in the stiffness matrix based on the conventional beam - column theory.

The computation of gradients of the objective function and constraints for such general formulations is usually performed by the finite difference scheme. However, the finite difference procedure involves a considerable amount of unnecessary and repetitive computations. Alternative approaches to

the problem of reanalysis of modified structures, and the efficient procedures developed and adopted for gradient computation in the present work are detailed in Chapter 4.

The formulations are studied using the examples of a tied cantilever and two cable stayed girders subjected to single load condition. Results are obtained for both the linear elastic behaviour and for the geometrically nonlinear behaviour of the structure. Also, some of the input design parameters are varied and the effect of the variation of these parameters on the optimum design are studied. The results of these studies are presented in Chapter 5 and the discussion of the results, the conclusions drawn and the suggestions for extension of this work are presented in Chapter 6.

1.3 REVIEW OF CURRENT LITERATURE IN THE RELATED TOPICS:

One of the first attempts at the optimum elastic design of framed structures was made by Brown and Ang |8|, who applied the gradient projection method to the design of frames. A continuous relation between the area and moment of inertia of wide flange beam members was determined, and each member was defined by a single design variable, its moment of inertia. They have shown that the minimum weight design under multiple loading conditions, is not a fully stressed design. Reinschmidt et al |9| have shown that the general iterative methods need not necessarily converge and even if they do, it need not

necessarily be to the minimum weight design. The successive linearisation approach was proposed by them for the design of frames. Romstad and Wang |10| solved the nonlinear programming problem by successive linearisation and application of linear programming.

Kavlie and Moe |11| and Moses and Onoda |12| have considered the problem of minimum weight design of elastic grillages. The problem is a nonconvex programming problem, which is solved by Kavlie and Moe by the penalty function approach.

Majid and Elliott |13| considered the optimum design of a gable frame subjected to deflection constraints, and solved the problem by a 'dynamic search method'. They have presented design charts for the optimum design of gable frames of arbitrary span and height subjected to the deflection specifications of BS 449. The formulation does not consider stress constraints. Majid and Anderson |14| formulated the problem of optimum design of general indeterminate structures using the force method of analysis. The constraints derived had separable variables, and they were linearised piecewise, and solved using the simplex method.

Kavlie and Moe |7| have employed the sequential unconstrained minimisation technique for the optimum design of framed ship structures subjected to different loading conditions.

Their formulation is very general and has been quite successfull; applied to the design of different structures. One formulation in the present work, namely the synthesis approach, has a certain amount of similarity with this work in its generality and the method of solution.

A different approach has also been tried by some authors. The conventional mathematical programming approaches to the optimal structural design suffer from the disadvantage that a number of analysis – design cycles is necessary to arrive at the optimum design. A possible way of reducing the computationa effort is to combine the mathematical programming methods with some iterative techniques. Wright and Feng |15| considered such an approach and termed it the 'multimode approach'. In this scheme, the stress ratio iteration method |16|, the basic iteration method |9| and the gradient projection method are used sequentially. The use of three different techniques sequentially results in the exploitation of the relative merits of each of these techniques to form an efficient composite method. Also, a detailed derivation of the behaviour gradient matrices is presented by them; these are used to compute the rates of change of the constraints with respect to the design variables. Davies and Wang |17| used a combination of iterative fully stressed design and linear approximation technique to obtain the optimum design of railway underframes, idealised as space frames.

The very challenging problem of optimization using discrete available sections has been considered by Cella and Logcher |18|. An overview of the algorithm for discrete optimization and its application to some practical design examples are presented by Cella and Soosaar |19|.

Cable stayed structures, especially for medium span bridges, have been quite popular in Europe for quite some time, and recently Leonhardt and Zellner |20| have shown that cable-stayed girder bridges are superior, in terms of economy and structural efficiency, to other types of bridges for all spans over 200 metres. The analysis of these bridges was considered by Stafford Smith |21| who used a mixed force-displacement approach, which he extended later |22| for the analysis of double plane cable stayed structures. Troitsky and Lazar |23| used a flexibility approach while Lazar |24| used a stiffness approach. It was observed that the designs exhibited geometrically nonlinear behaviour, which was considered by Lazar |24| who used a Newton-Raphson type solution procedure. Tang |25| used a transfer matrix technique for the analysis, including the nonlinear terms in the form of imaginary external loads. Kajita and Cheung |26| performed the linear static and dynamic analysis of the structure by the finite element method, considering the deck as a plate and the tower as a beam. Morris |27| performed the dynamic analysis of the planar structure both under

linear and nonlinear behaviour, and found that the effects of the nonlinearities were marginal.

An approach to the efficient design of these structures is presented by Lazar et al |28| who present a kind of load balancing approach in which the cable pretensions (or post-tensions as they call it) are chosen in such a way that the maximum bending moment along the girder is minimised by an iterative technique using the flexibility method. It may be mentioned that the choice of minimum design bending moment along the girder as the optimality or efficiency criterion is rather dubious. Since the structure is statically indeterminate, the properties of the sections would have to be determined before the analysis. The formulations considered in the present work include the tensioning force as an additional design variable, thereby controlling the behaviour of the structure. Tang |29| highlights some of the problems encountered in the practical design of these structures. However, no effort seems to have gone to seek the optimal design of this class of structures.

CHAPTER TWO

THE PROBLEM FORMULATION

2.1 INTRODUCTION:

The general problem considered in the present work is the optimum design of planar steel frames with linear and nonlinear elastic behaviour under single loading condition, with special reference to cable stayed planar frames. Both joint and member loads are considered. Provision is made for different types of member cross section.

The different types of members considered in the programs are: a. Box beam, b. I-Section, c. Latticed section of triangular or square shape, and d. High tensile cable. Any other structural section can be included with little effort. For each type of member there are some preassigned parameters which remain constant during the process of optimization and some variable parameters which form part of the design vector. The members are assumed to be prismatic. Stress constraints are imposed on each member and lower bounds on each variable. A list of preassigned parameters, design variables, expressions for the cost of the member, and the stress constraints considered for each member type included in the computer program is given in Table 2.1.

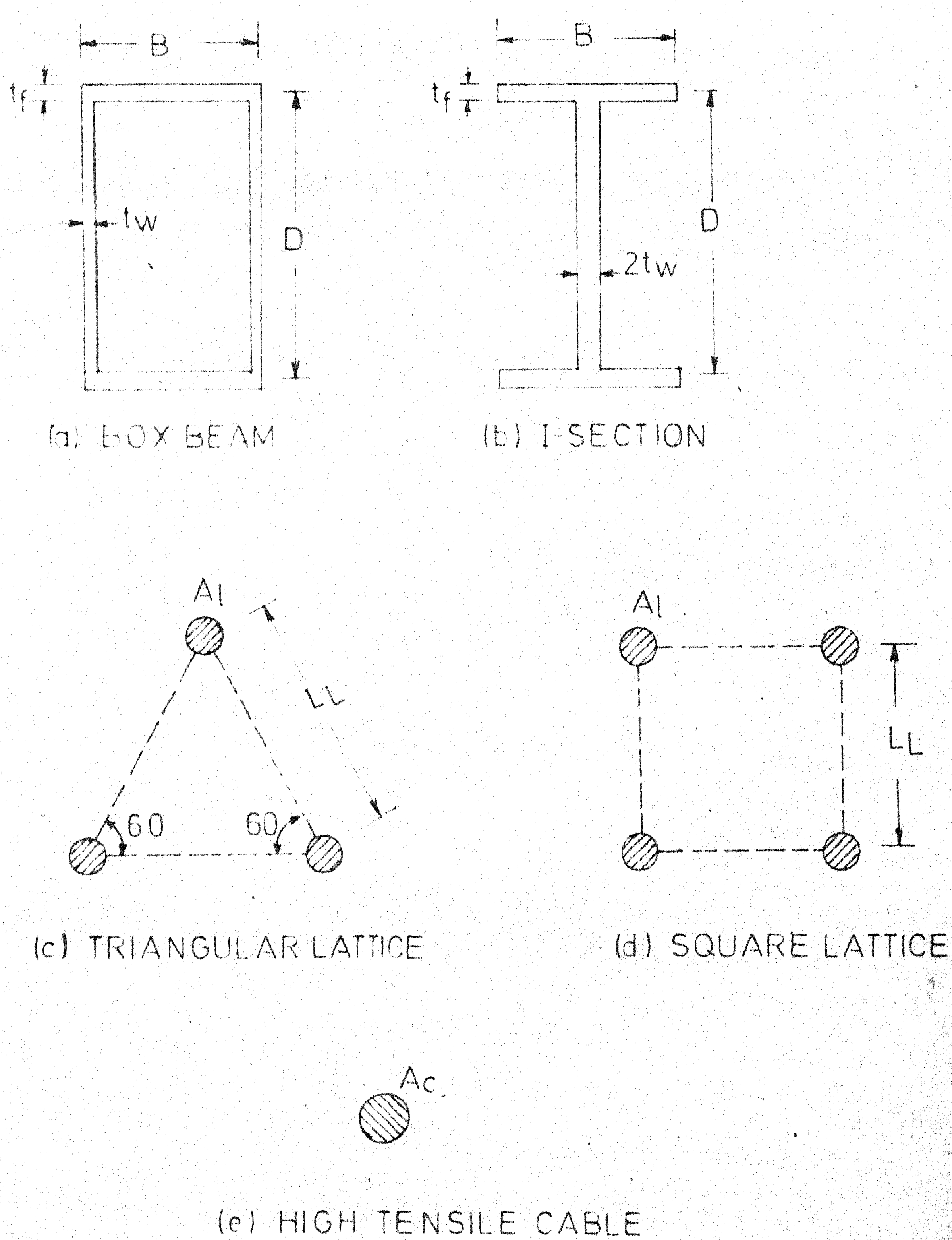


FIG. 2.1 TYPICAL CROSS SECTIONS CONSIDERED IN THE PRESENT WORK

Table 2.1: Preassigned Parameters, Design Variables, Constraints and Costs of Different Types of Members.

Member Type	Preassigned Parameters	Design Variables	Cost	Stress Constraints
1. Box beam and I-Section	1. Depth of beam (D) 2. Width of flange (B)	1. Thickness of Web (t_w) 2. Thickness of flange (t_f)	$2CL\ (Bt_f + Dt_w)$	1. Maximum stress in flange due to combined bending and axial force. 2. Maximum average shear stress in the web.
2. Latticed columns				
a. Triangular	Side of Triangle (LL)	Area of each leg (A_1)	$3CL\ A_1$	Maximum stress in leg due to combined bending and axial force.
b. Square	Side of Square (LL)	-do-	$4CL\ A_1$	-do-
3. High Tensile Cable	-	1. Area of cable (A_c) 2. Pretension force (P)	$CL\ A_c$	1. Stress due to pretension force 2. Stress due to pretension and external loading.

C = Cost per unit volume of member

L = Length of the member.

The objective function chosen is the cost of materials. The total cost is computed as the sum of the costs of constituent members. The cost of a member is simply the volume times the cost per unit volume of material for that member. The inclusion of fabrication and erection costs of each member, and the costs of additional structural components like the web stiffeners of beams and bracings of latticed members can be admitted in an indirect manner. This is simply achieved by increasing the unit cost of the member by a certain percentage.

From Table 2.1 it may be observed that the objective function is a linear function of the design variables for the different cross sections considered. The depth of the box beam or I-section is considered as a preassigned parameter because it is quite often determined from nonstructural considerations. Furthermore, for large span bridge sections, shapes are determined from aerodynamic stability considerations, and this results in fixed values of the width of flange and depth of the idealised cross section.

2.2 THE SYNTHESIS APPROACH

2.2.1 Analysis:

This formulation uses the stiffness method of analysis. The stiffness matrix of the structure is generated at every

design point during the search procedure, and the equilibrium equations are solved to obtain the joint displacements. The nonlinear effects are embedded in the stiffness matrix formulation as described in Chapter 3. Once the joint displacements are known the maximum forces in the members are evaluated and then the maximum stresses are computed. The relevant equations are given below:

The equilibrium equations:

$$K \vec{u} = \vec{p} \tag{2.1}$$

in which,

K is the stiffness matrix of the structure,

$\vec{u}$ is the nodal displacement vector, and

$\vec{p}$ is the nodal force vector.

Member end forces for member i:

$$\vec{f}_i = S_i \, T_i \, \vec{\delta}_i + \vec{ff}_i \tag{2.2}$$

in which,

$\vec{f}$ is the vector of member end forces in the element coordinate system,

S is the stiffness matrix of the member in the element coordinate system,

T is the coordinate transformation matrix,

$\vec{\delta}$ is the vector of joint displacements of the two ends of the member in the global coordinate system,

$\vec{ff}$ is the vector of fixed end forces in the member in the element coordinate system,

and the subscript i denotes that these quantities are related to member i.

The maximum bending moment, if any, along the length of the member is computed from statics. The design bending moment, shear force and axial force for the member are then chosen and the constraints evaluated.

2.2.2 Constraints:

The maximum fibre stress in combined bending and axial force is given by,

$$\sigma_i = \left| \frac{Q_i}{A_i} \right| + \left| \frac{M_i}{Z_i} \right| \tag{2.3}$$

in which,

σ is the maximum fibre stress,

Q is the axial force,

A is the area of cross section,

M is the design bending moment,

Z is the section modulus of the member, and

the subscript i denotes that these quantities are related to member i.

The average shear stress in the web is given by,

$$q_i = \frac{V_i}{Aw_i} \tag{2.4}$$

in which,

q is the average shear stress,

V is the design shear force, and

Aw is the area of the web.

The stress constraints are written in the form

$$1 - (\sigma_i/\sigma_{all,i}) \geq 0 , \tag{2.5}$$

and

$$1 - (q_i/q_{all,i}) \geq 0 \tag{2.6}$$

$\sigma_{all,i}$ and $q_{all,i}$ are the allowable maximum fibre stress and shear stress respectively for member i. Normalization of the constraints, as indicated in Equations 2.5 and 2.6, facilitates a smooth behaviour of the optimization problem. The constraints would always lie between 0 and 1, and the value of nearly 1 for a constraint shows that the corresponding stress is nearly zero in the member.

For high tensile cables, different sets of constraints are used. The limitation on the pretension stress is given by

$$1 - \left(\frac{P_i}{A_i} \right) / (0.6\sigma_{yc}) \geq 0 \tag{2.7}$$

in which,

P is the pretension force in the cable, and

σ_{yc} is the yield stress or the proof stress of the material of the cable.

The limitation on the final stress in the cable is given by,

$$1 - \left(\frac{Q_i}{A_i}\right) / (0.8\ \sigma_{yc}) \geq 0 \qquad (2.8)$$

Displacement constraints, if any, are appended to the above constraint set.

2.2.3 The Optimization Problem:

The design vector in this formulation consists of the member dimensions and pretension forces. The formulation results in the following mathematical programming problem:

$$\text{Find} \quad \vec{x} \equiv \vec{d} \quad \text{to minimise} \quad f(\vec{d}) = \sum_{j=1}^{NM} C_j$$

$$\begin{aligned} \text{Subject to} \quad & g_j(\vec{d}) \geq 0, \quad j = 1, \ldots, NC \\ & d_i \geq d_i^l, \quad i = 1, \ldots, NDV \\ & u_k^l \leq u_k \leq u_k^u \text{ for } k \in M \end{aligned} \qquad (2.9)$$

in which,

$\vec{x}$ is the general design variable vector of dimension N,

$\vec{d}$ is the subset of $\vec{x}$ containing elements corresponding to member dimensions and pretension forces of dimension NDV. In this formulation $\vec{x} \equiv \vec{d}$, and N = NDV.

C_j is the cost of the j-th member,

NM is the number of members,

g_j is the j-th behaviour constraint,

NC is the number of behaviour constraints,

d_i^l is the lower bound on the i-th design variable,

u_k^l and u_k^u are respectively the upper and lower bounds on the k-th displacement, and

M is the number of degrees of freedom of the structure.

A flow diagram of the procedure for evaluation of the objective function and constraints in this formulation is given in Fig. 2.2.

2.3 THE INTEGRATED SYNTHESIS APPROACH:

The synthesis approach requires that the structure be analysed by solving the equilibrium equations at every design point during the search procedure. This consumes a considerable portion of the computational effort. The only way to avoid the solution of simultaneous equations so many times is to consider the displacements of joints as additional design variables and to treat the equilibrium equations as equality constraints. Such a formulation, of course, increases the number of variables and constraints and introduces the troublesome nonlinear equality constraints to the optimization problem. The purpose behind the study of this formulation is to compare the efficiencies of the two formulations. It may not be out of place to mention here that a similar formulation for the optimum design of truss structures was made by Schmit and Fox |30,31|.

The design variable vector, therefore, consists of two parts - the joint displacements, $\vec{u}$, and member dimensions, $\vec{d}$.

$$\therefore \quad \vec{x} \equiv \left\{ \begin{array}{c} \vec{u} \\ \hline \vec{d} \end{array} \right\} \tag{2.10}$$

2.3.1 <u>Constraints</u>:

The inequality constraints on stresses are computed from the displacements in the same manner as detailed in Section 2.2.2. The equality constraints on equilibrium are written as,

$$K \vec{u} - \vec{p} = \vec{0} \tag{2.11}$$

The elements of the stiffness matrix K are linear functions of the dimension variables $\vec{d}$. The constraints given by Eq. 2.11 are, therefore, nonlinear equality constraints as products of dimension variables and displacement variables appear in these equations. To effect smooth behaviour of the optimization procedure, each of the equations is normalised with respect to a preselected force value. Let this value be represented by $FMAX_k$ for the k-th equilibrium equation. The k-th normalised equality constraint therefore becomes,

$$l_k (\vec{x}) = \frac{1}{FMAX_k} \left[\sum_{r=1}^{M} K_{kr} u_r - p_k \right] = 0 \tag{2.12}$$

The preselected normalising force and moment for the structure are fed in the computer program and the value $FMAX_k$, is picked from these two depending upon whether the k-th equation expresses the force equilibrium or the moment equilibrium.

2.3.2 <u>The Optimization Problem</u>:

This problem thus results in the following nonlinear programming problem:

$$\text{Find } \vec{x} \equiv \left\{ \begin{array}{c} \vec{u} \\ \hline \vec{d} \end{array} \right\} \text{ to minimise } f(\vec{d}) = \sum_{j=1}^{NM} C_j$$

$$\begin{aligned} \text{Subject to} \quad & g_j(\vec{x}) \geq 0, \quad j = 1, \ldots, NC \\ & l_k(\vec{x}) = 0, \; k = 1, \ldots, M \\ & d_i \geq d_i^l, \quad i = 1, \ldots, NDV \\ & u_k^l \leq u_k \leq u_k^u, \; k \in M \end{aligned} \tag{2.13}$$

In this formulation the total number of design variables $N = NDV + M$.

A flow diagram of the procedure for evaluation of the objective function and constraints in this formulation is given in Fig. 2.3.

It may be mentioned here that the intermediate designs obtained during the optimization procedure in this formulation will not be physically meaningful, since they would not satisfy the equilibrium equations.

2.4 THE FULLY CONSTRAINED DESIGN FORMULATION:

The optimal design results of the cable stayed planar frames studied by the above two formulations indicate that the optimum design of this class of structures is a 'fully constrained

design', that is, every design variable defining the cross sectional property of the elements takes a value at the optimum design such that it either satisfies a stress constraint or is bounded by the side constraint. In a box beam or I-Section, the thickness of the web is governed either by the minimum thickness constraint or by the shear stress constraint. The thickness of the flange, likewise, is governed either by the extreme fibre stress constraint or by the minimum thickness constraint. In the high tensile cable, the area is governed either by the allowable pretension stress or by the allowable final (or working) stress. Thus the total number of active constraints at the optimum is at least equal to the number of dimension variables defining the cross sections of the members. The pretension forces in the cables, however, are the only free parameters which influence the cost once the constraints are satisfied.

The iterative procedure of a 'fully constrained design' in which members are resized by the satisfaction of the constraints suggests itself as an efficient design procedure. The pretension forces can be taken as design variables in an optimization formulation to minimise the cost of materials. The design would be performed as a 'fully constrained design' at every design point. The behaviour constraints and the side constraints are implicitly satisfied in this approach and hence the resulting optimization problem is an unconstrained minimization problem.

2.4.1 Design:

The relevant equations used for the design of components by the fully constrained design procedure are given below:

a. Box beam and I-Section:

The area of the web is given by $A_w = 2\,Dt_w$ (2.14)

To satisfy the shear stress constraint, the required web thickness is,

$$t_{w1} = \frac{V}{q_{all} \cdot 2D} \tag{2.15}$$

Therefore, the thickness of the web is chosen as

$$t_w = \mathrm{MAX}\,(t_{w1},\ t_{wmin}) \tag{2.16}$$

where t_{wmin} is the minimum allowable thickness of the web.

The area of cross section is given by,

$$A = 2(Bt_f + Dt_w) \tag{2.17}$$

and the section modulus is,

$$Z = D\,(Bt_f + Dt_w/3) \tag{2.18}$$

The combined bending and axial stress is given by Equation 2.3. Since t_w is known, the required thickness of the flange t_{f1} is obtained by satisfying this stress constraint, viz.,

$$\sigma_{all} = \left| \frac{Q}{2(Bt_{f1} + Dt_w)} \right| + \left| \frac{M}{D(Bt_{f1} + Dt_w/3)} \right| \tag{2.19}$$

The thickness of the flange is then chosen as

$$t_f = \text{MAX}\,(t_{f1},\ t_{fmin}) \tag{2.20}$$

where t_{fmin} is the minimum allowable thickness of the flange.

b. <u>High Tensile Cable</u>:

The area of the cable is obtained from,

$$A_c = \text{MAX}\left(\frac{P}{0.6\,\sigma_{yc}},\ \frac{Q}{0.8\,\sigma_{yc}},\ A_{cmin}\right) \tag{2.21}$$

where A_{cmin} is the minimum allowable area of the cable.

c. <u>Triangular Lattice</u>:

The required area of each leg of the lattice, A_{L1}, is found from the satisfaction of the combined axial and bending stress,

$$\sigma_{all} = \left|\frac{Q}{3A_{L1}}\right| + \left|\frac{2M}{\sqrt{3}A_{L1}\ LL}\right| \tag{2.22}$$

in which LL is the lateral distance between the legs of the ~~ka~~ lattice. The area of the leg is chosen as,

$$A_l = \text{MAX}\,(A_{L1},\ A_{Lmin}) \tag{2.23}$$

where A_{Lmin} is the minimum allowable area of the leg.

d. <u>Square Lattice</u>:

The required area of the leg in this case is found from

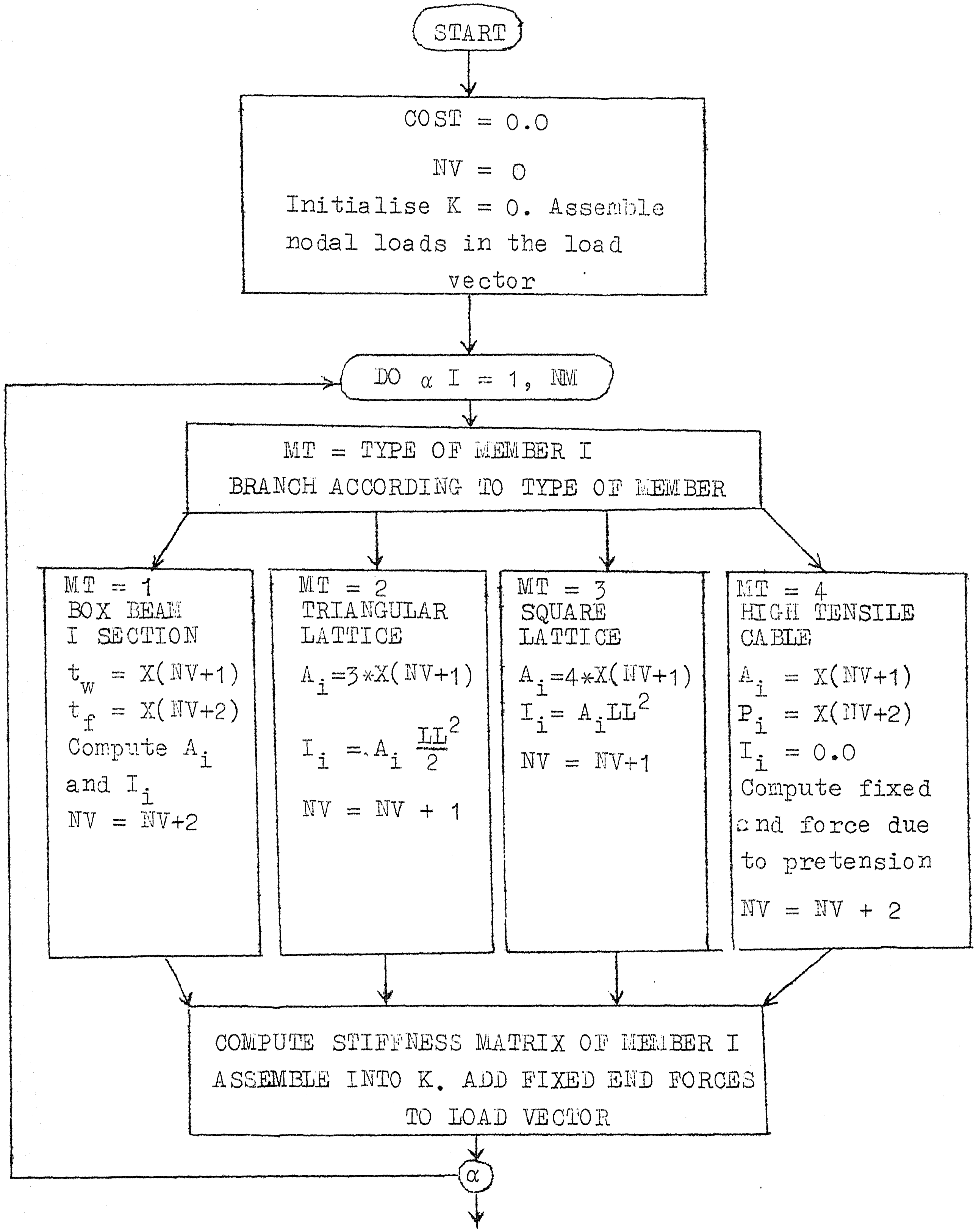
$$\sigma_{all} = \left|\frac{Q}{4A_{L1}}\right| + \left|\frac{M}{2A_{L1}.LL}\right| \tag{2.24}$$

and the area of the leg is chosen using Equation 2.23.

2.4.2 The Optimization Problem:

The fully constrained design leading to the minimum cost design formulation, therefore, results in an unconstrained minimization problem, with the iterative fully constrained design performed at every stage. Thus this formulation can be considered a combination of the mathematical programming and optimality criterion techniques. A detailed flow chart of the optimization process is given in Fig. 2.4 and a flow chart for the design is given in Fig. 2.5.

This formulation cannot directly handle displacement constraints. This is because the optimum design of a structure with active displacement constraints is, in general, not fully constrained in the sense defined earlier.


START
COST = 0.0
NV = 0
Initialise K = 0. Assemble nodal loads in the load vector
DO α I = 1, NM
MT = TYPE OF MEMBER I
BRANCH ACCORDING TO TYPE OF MEMBER
MT = 1
BOX BEAM
I SECTION
t_w = X(NV+1)
t_f = X(NV+2)
Compute A_i and I_i
NV = NV+2
MT = 2
TRIANGULAR LATTICE
A_i=3*X(NV+1)
$I_i = A_i \frac{LL^2}{2}$
NV = NV + 1
MT = 3
SQUARE LATTICE
A_i=4*X(NV+1)
$I_i = A_i LL^2$
NV = NV+1
MT = 4
HIGH TENSILE CABLE
A_i = X(NV+1)
P_i = X(NV+2)
I_i = 0.0
Compute fixed end force due to pretension
NV = NV + 2
COMPUTE STIFFNESS MATRIX OF MEMBER I
ASSEMBLE INTO K. ADD FIXED END FORCES TO LOAD VECTOR
α

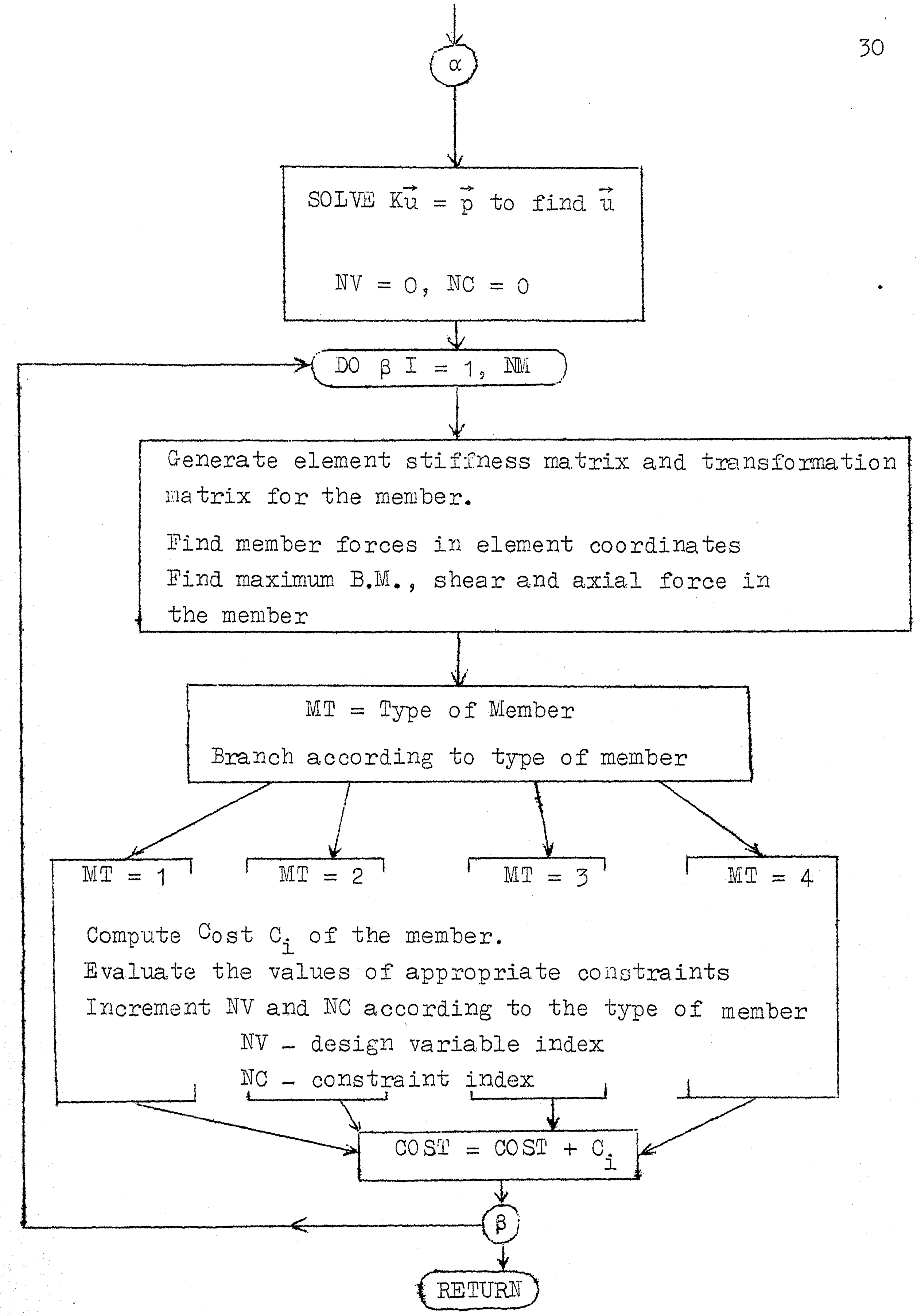
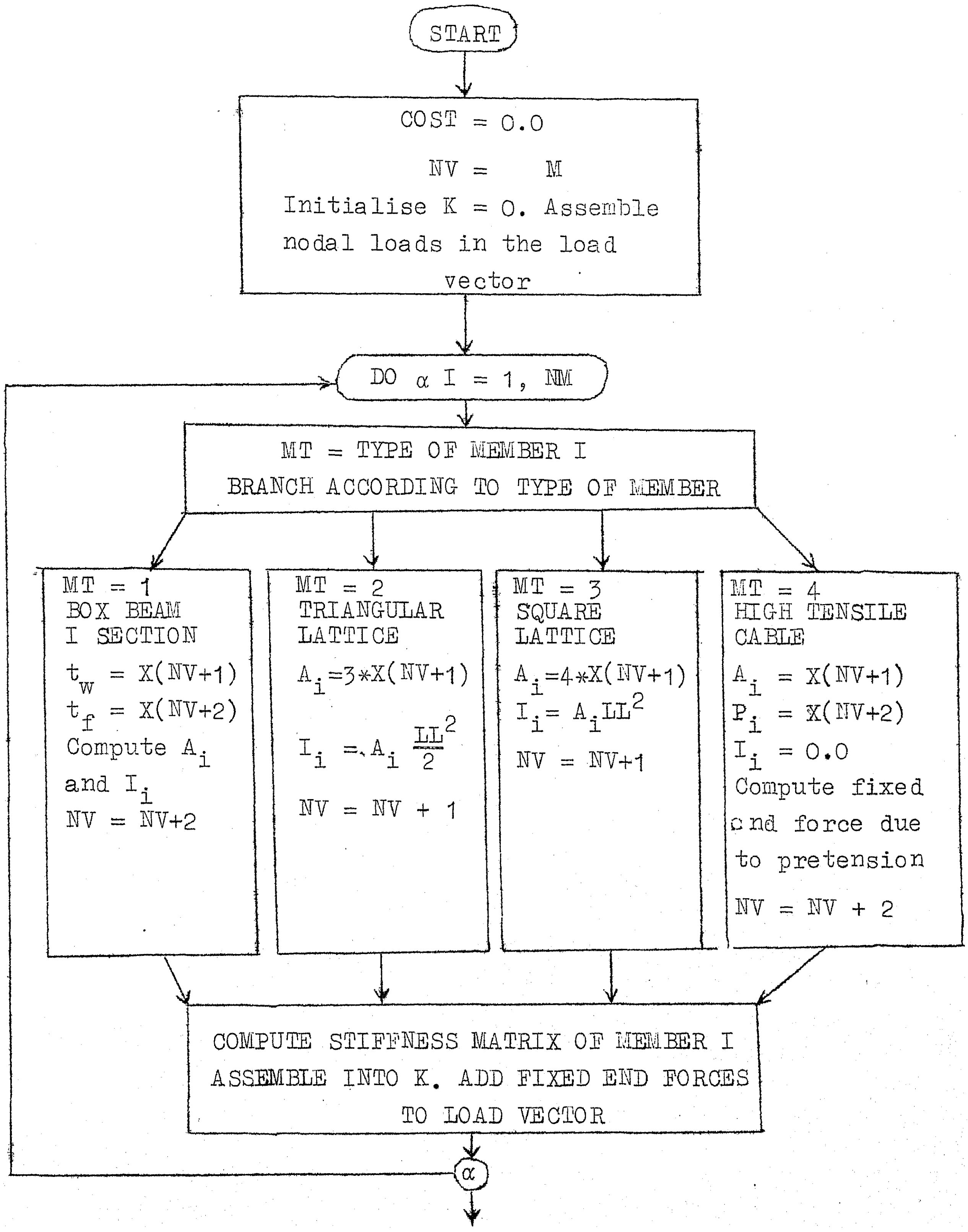


Fig. 2.2: Flow Chart for Evaluation of Objective Function and Constraints-Synthesis Formulation.


START
COST = 0.0
NV = M
Initialise K = 0. Assemble nodal loads in the load vector
DO α I = 1, NM
MT = TYPE OF MEMBER I
BRANCH ACCORDING TO TYPE OF MEMBER
MT = 1
BOX BEAM
I SECTION
t_w = X(NV+1)
t_f = X(NV+2)
Compute A_i and I_i
NV = NV+2
MT = 2
TRIANGULAR
LATTICE
A_i=3*X(NV+1)
$I_i = A_i \frac{LL^2}{2}$
NV = NV + 1
MT = 3
SQUARE
LATTICE
A_i=4*X(NV+1)
$I_i = A_i LL^2$
NV = NV+1
MT = 4
HIGH TENSILE
CABLE
A_i = X(NV+1)
P_i = X(NV+2)
I_i = 0.0
Compute fixed end force due to pretension
NV = NV + 2
COMPUTE STIFFNESS MATRIX OF MEMBER I
ASSEMBLE INTO K. ADD FIXED END FORCES TO LOAD VECTOR
α

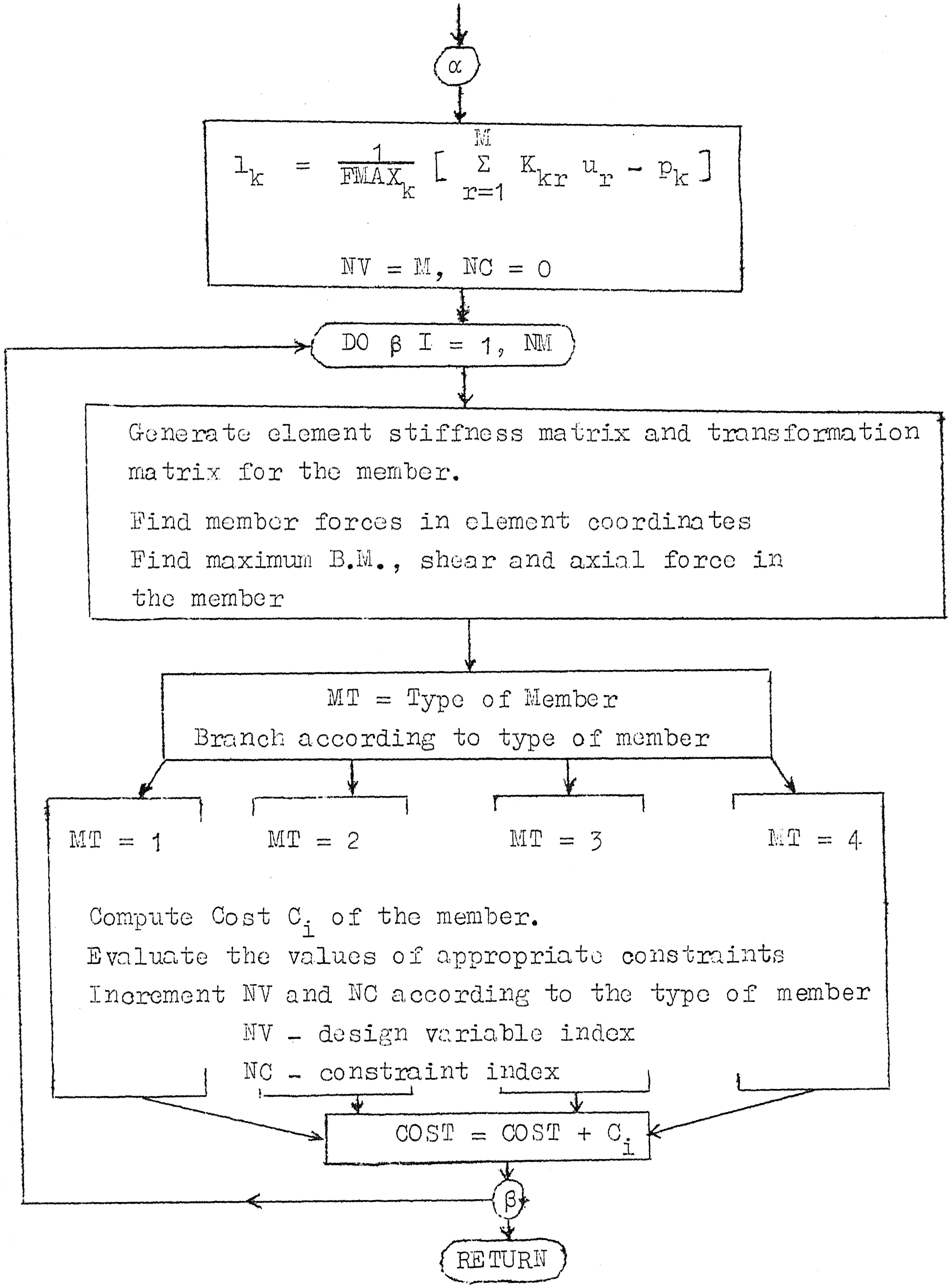


Fig. 2.3: Flow Chart for Evaluation of Objective Function and Constraints - Integrated Synthesis Approach

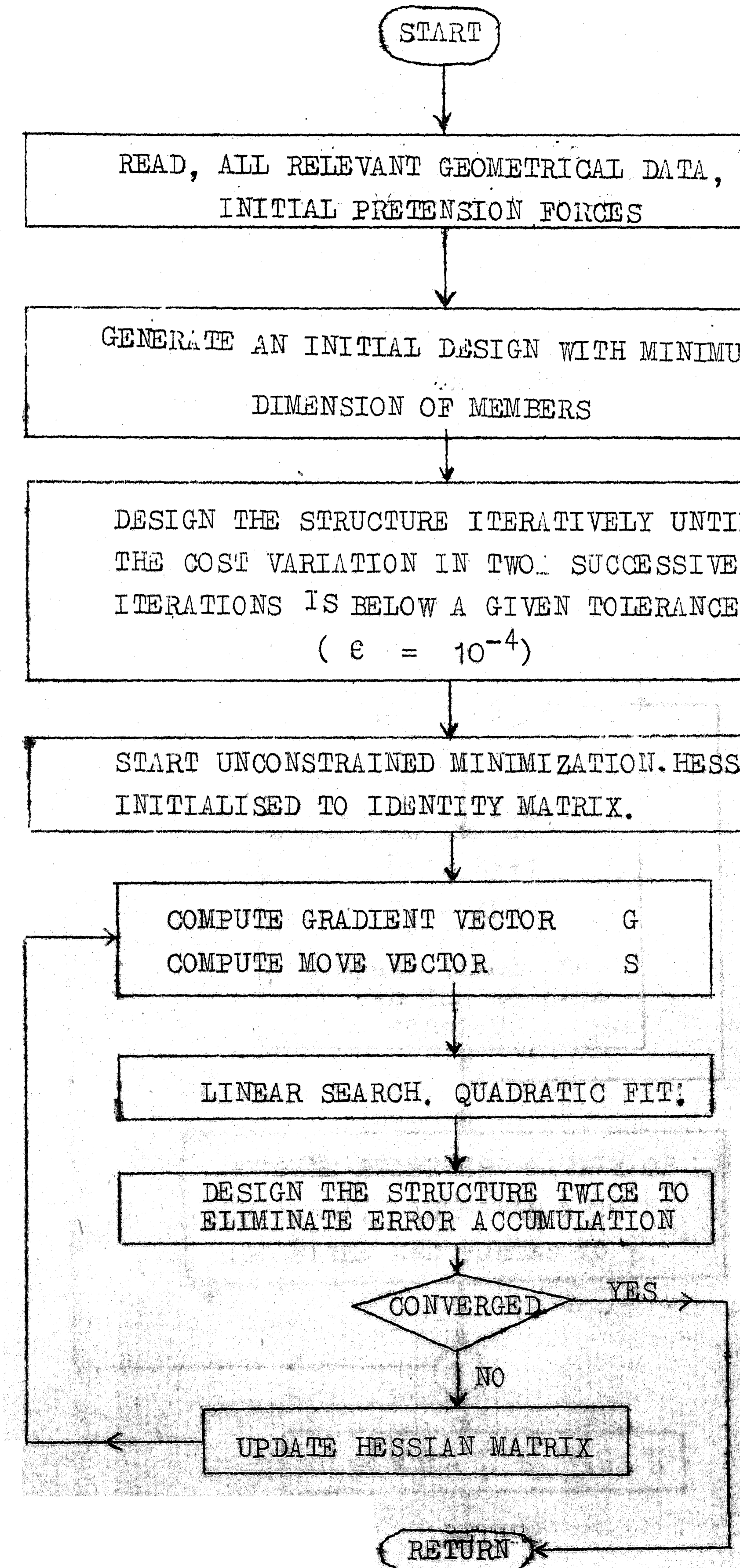
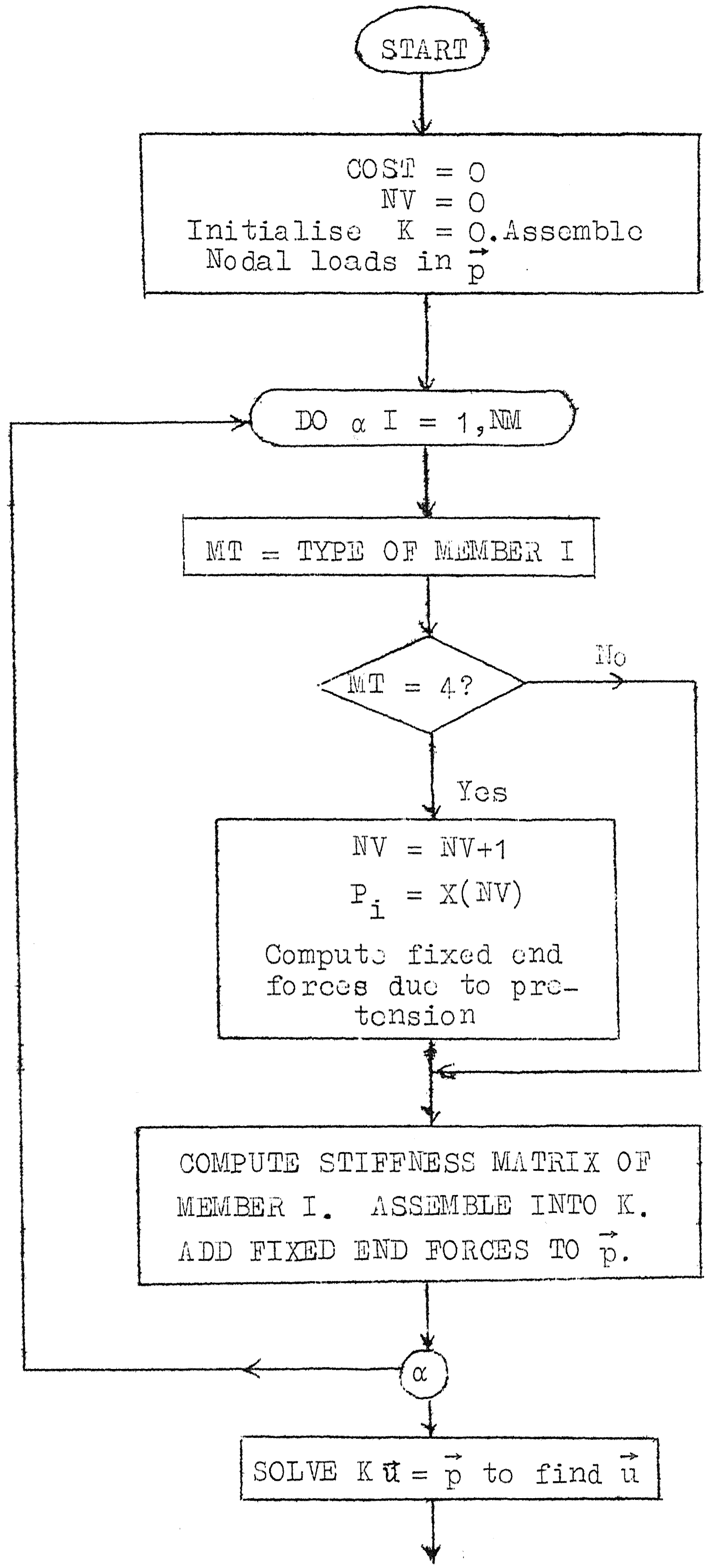


Fig. 2.4: Flow chart for optimization by the fully constrained design formulation.


START
COST = 0
NV = 0
Initialise K = 0. Assemble
Nodal loads in $\vec{p}$
DO α I = 1,NM
MT = TYPE OF MEMBER I
MT = 4?
No
Yes
NV = NV+1
P_i = X(NV)
Compute fixed end forces due to pre-tension
COMPUTE STIFFNESS MATRIX OF MEMBER I. ASSEMBLE INTO K. ADD FIXED END FORCES TO $\vec{p}$.
α
SOLVE $K\vec{u} = \vec{p}$ to find $\vec{u}$

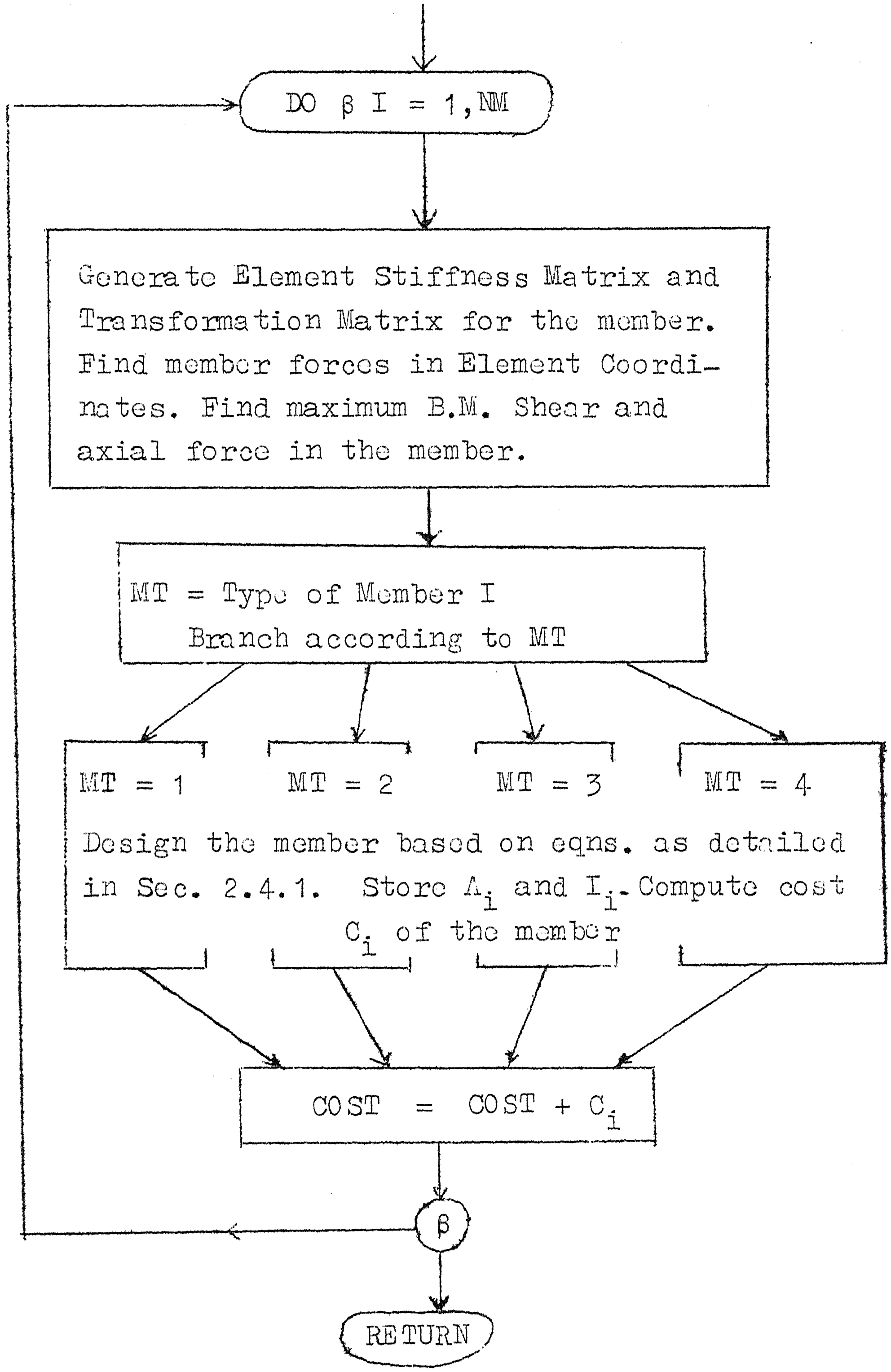


Fig. 2.5: Flow Chart for Design and Evaluation of Objective Function - Fully Constrained Design Formulation.

CHAPTER THREE

NONLINEAR ANALYSIS

3.1 INTRODUCTION:

The nonlinear behaviour of structural systems is broadly classified into three categories: (i) Geometric Nonlinearity, which arises from the nonlinear terms in the kinematic equations, (ii) Material Nonlinearity, which arises from the nonlinearity in the constitutive equations, and (iii) Combined Material and Geometric Nonlinearity.

The structures considered in the present study exhibit only geometric nonlinearity, since at the working load the material of the structure is assumed to be linearly elastic. These structures are supported by high tensile cables with a certain amount of initial tension and the flexural elements are subjected to a significant amount of axial force. The presence of axial force reduces the flexural stiffness of these elements and increases the fixed end moments. This interaction between the axial load and bending is commonly known as the beam-column effect and is the most important reason for the nonlinear behaviour of these structures. A certain contribution to the nonlinearity is also made by the large deformation of

structures, in which case the equilibrium equations should be written in the deformed configuration of the structure. This effect, however, is very small and the deformations of the structures considered in the present work do not warrant the inclusion of this effect. The high tensile cables sag due to self weight and their behaviour is also nonlinearly elastic |20|. This nonlinearity is significant only for cables of very large spans and when the stress level in the cables is low (Figure 6 of Ref. 20). The optimum design in the problems considered in the present work results in the pretension and final stress levels to be relatively high and this justifies that the nonlinearity due to cable sag be neglected.

A brief review of the techniques of nonlinear analysis of structures is presented in this chapter. The choice of the 'Secant Stiffness Matrix' for the beam elements used in the present work is discussed, and the modifications to the computer programs necessary to incorporate the nonlinear analysis are also outlined.

3.2 NUMERICAL TECHNIQUES FOR NONLINEAR STRUCTURAL ANALYSIS:

The techniques of nonlinear analysis followed the methods of linear analysis and naturally nonlinearities were attempted as corrections to the linear solutions. The methods developed from these were generally iterative, incremental or a combination of both. Fig. 3.1 shows the Newton-Raphson or the 'Tangent Stiffness'

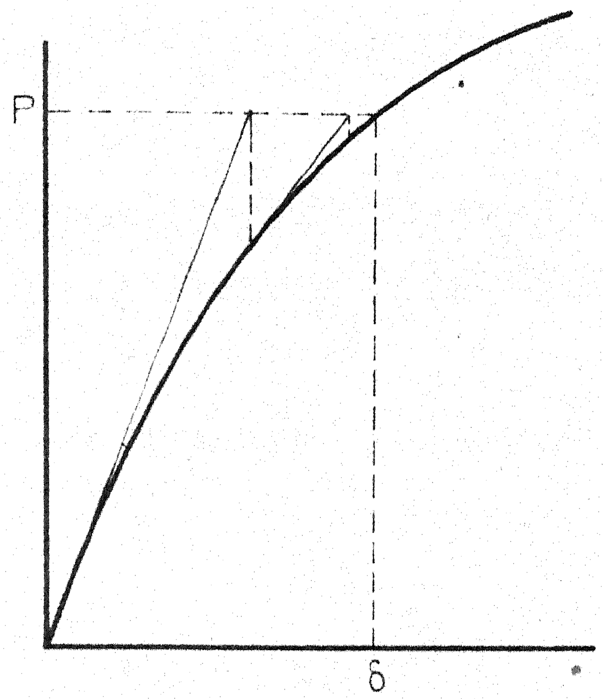


FIG. 3·1 TANGENT STIFFNESS APPROACH

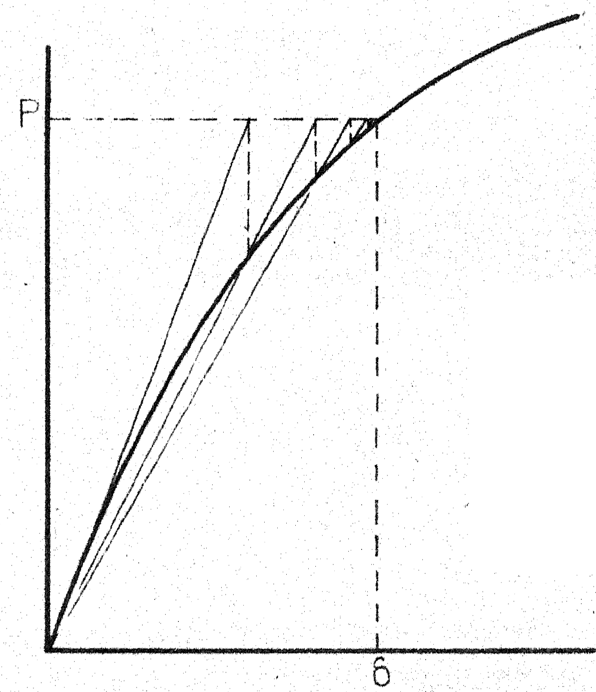


FIG. 3·2 SECANT STIFFNESS APPROACH

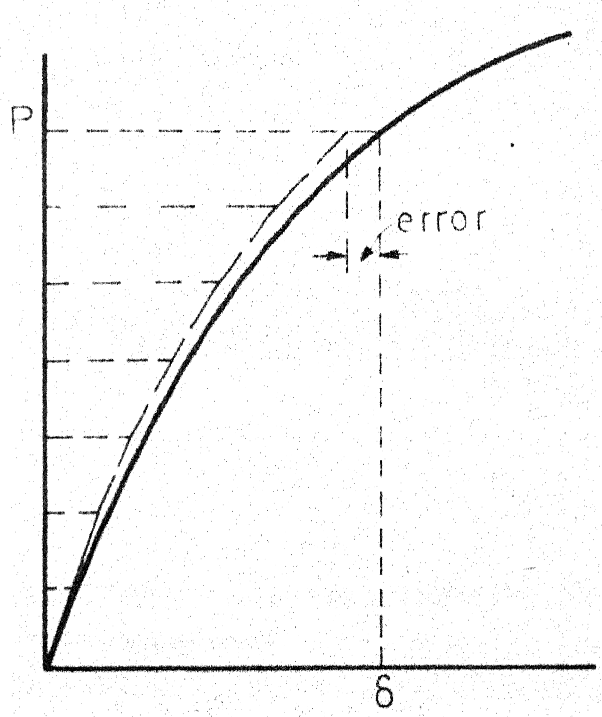


FIG. 3·3 PURELY INCREMENTAL APPROACH

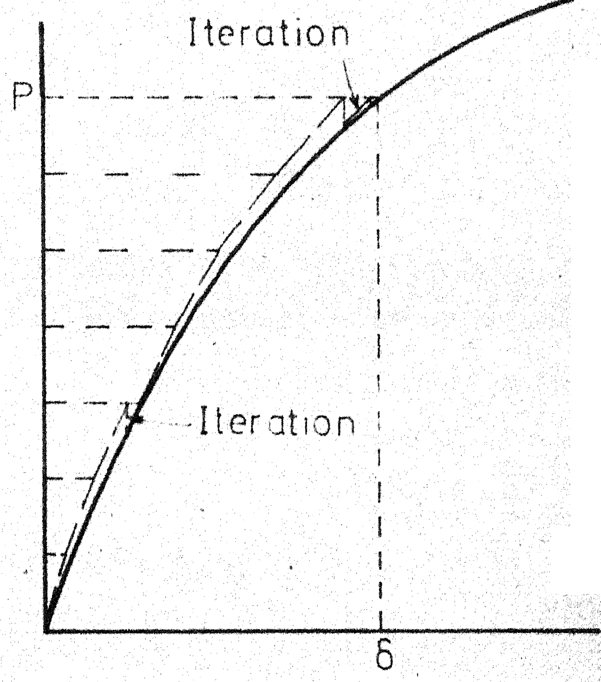


FIG. 3·4 INCREMENTAL-CUM-ITERATIVE APPROACH

METHODS OF NONLINEAR STRUCTURAL ANALYSIS

approach to a typical single degree of freedom problem in structural analysis. Fig. 3.2 shows the direct or 'Secant Stiffness' approach to the same problem. Both are iterative methods of analysis. The tangent stiffness approach is more efficient, in terms of rate of convergence, than the secant stiffness approach. Fig. 3.3 shows the purely incremental technique of solving the same problem. In this approach the error in the solution increases with the increase in the load. This is corrected by incorporating the tangent stiffness iterations after every few increments. Such an incremental-cum-iterative approach, shown in Fig. 3.4, is the most efficient and accurate technique for studying the nonlinear load-deformation history of a structure.

Another type of corrective approach to the linearised equations is the 'initial stress' or 'geometric stiffness' matrix. This arises due to the effects of the initial stress during an increment performing work on the incremental changes in the nodal displacements of the element. This approach is entirely satisfactory for the classical stability analysis of structures with negligible prebuckling deformations. However, it is insufficient for the calculation of general geometrically nonlinear behaviour |32|.

Other efficient methods for tracing the complete load-deflection history of highly nonlinear structures are based

on the parametric differentiation method. Self correcting |33| and numerical integration methods to solve the resulting initial value problem have been used by various investigators.

Analysis of nonlinear structures can also be performed by the direct minimization of the potential energy functional |34,35|. This approach, however, suffers from the disadvantage that proper scaling of variables in the analysis problem is required to ensure fast and accurate convergence. Computationally, this method is not competitive with the methods detailed above.

An excellent comprehensive assessment of the solution techniques applicable to static nonlinear structural analysis is presented by Tillerson, Stricklin and Haisler |36|.

3.3 NONLINEAR ANALYSIS APPROACH IN THE PRESENT WORK:

In the search procedures for optimization, a sequence of designs is generated such that the objective function decreases monotonically. Let $\vec{d}_i$ and $\vec{d}_{i+1}$ be two consecutive design points. At point i the equilibrium equations are given by,

$$[K(\vec{d}_i, \vec{u}_i)]\ \vec{u}_i = \vec{p}_i \tag{3.1}$$

which, let us say, have been solved to find $\vec{u}_i$. At point (i+1) the equilibrium equations are given by

$$[K(\vec{d}_{i+1}, \vec{u}_{i+1})]\ \vec{u}_{i+1} = \vec{p}_{i+1} \tag{3.2}$$

The load vector also changes from point to point because the fixed end moments also depend on the axial load in the non-linear analysis.

$$\text{Let} \qquad \vec{u}_{i+1} = \vec{u}_i + \Delta\vec{u} \tag{3.3}$$

For the tangent stiffness approach we require the approximate estimate of the displacements and the unbalanced forces under these estimated displacements. Let the approximate estimate of the displacements be $\vec{u}_i$. The unbalanced forces at point (i + 1) under these displacements are given by,

$$\Delta\vec{p} = \vec{p}_{i+1} - [K(\vec{d}_{i+1}, \vec{u}_i)] \vec{u}_i \tag{3.4}$$

The tangent stiffness equations are

$$[TK(\vec{d}_{i+1}, \vec{u}_i)] \Delta\vec{u} = \Delta\vec{p} \tag{3.5}$$

in which TK is the tangent stiffness matrix at point i+1 evaluated using $\vec{u}_i$ as the estimated displacements. $\Delta\vec{u}$ is found by solving the Equations 3.5, and thus $\vec{u}_{i+1}$ is obtained. Thus in this approach it is necessary to evaluate two stiffness matrices, K to find the unbalanced forces and TK to find the corrective displacements.

In the direct approach we assume that the design points i and i+1 are sufficiently close, so that the displacements $\vec{u}_i$ and $\vec{u}_{i+1}$ would also not be very different. In such a case we may permit the approximation that

$$K(\vec{d}_{i+1}, \vec{u}_{i+1} \simeq K(\vec{d}_{i+1}, \vec{u}_i) \tag{3.6}$$

Equation 3.2 may be then approximated to

$$[K(\vec{d}_{i+1}, \vec{u}_i)]\ \vec{u}_{i+1} = \vec{p}_{i+1} \tag{3.7}$$

The stiffness matrix is computed and Equation 3.7 solved to obtain $\vec{u}_{i+1}$ directly. This approach requires the assembly and inversion of one matrix, and is sufficiently accurate if the nonlinearities are small and the points are quite close. It is known that the structures considered in the present work exhibit only a small amount of nonlinearity. It is, therefore, decided that the direct formulation, as given by Equation 3.7 would be sufficiently accurate for embedment in the optimization procedure. Furthermore, if the analysis is performed iteratively at the beginning of every linear search in the optimization procedure, any error in the displacements accumulated over the previous linear search would be eliminated. The iterative equations are given by,

$$[K(\vec{d}, \vec{u}^{(q)})]\ \vec{u}^{(q+1)} = \vec{p}^{(q+1)} \tag{3.8}$$

in which the superscript in parentheses denotes the iteration count.

The beam element may be modelled either by the finite element approach |37-45| or by the stiffness approach using the conventional beam-column theory |46-54|. The finite element approach using the generalised polynomial requires numerical integration to be performed to obtain the nonlinear stiffness matrix. The nonlinear stiffness matrix based on the

beam-column theory is the easiest to program, and requires very little extra computational effort over the linear stiffness matrix. In fact the linear stiffness matrix is a particular case of the nonlinear stiffness matrix. The best choice of the stiffness matrix is, therefore, the secant stiffness matrix based on the conventional beam-column theory. The stiffness matrix is given in Appendix A.

It may be mentioned here that Equation 3.2 is directly used in the constraint set of the integrated synthesis approach, because in this formulation both $\vec{d}$ and $\vec{u}$ are the design variables. Thus the nonlinear equality constraints do not involve any approximation whatsoever.

CHAPTER FOUR

THE METHOD OF OPTIMIZATION

4.1 CHOICE OF THE MÉTHOD:

The formulations of the optimal design problem considered in the present work lead to nonlinear programming problems with linear objective function and linear and nonlinear constraints. One of the objectives of the present work is to compare the efficiencies of the first two formulations described in Chapter 2. This comparison should be done with respect to one optimization program. The three general classes of widely used nonlinear programming methods are: a) The Gradient Projection Method of Rosen |55| modified subsequently by Goldfarb |56|. This works well with linear constraints. However, the efficiency is considerably reduced in the case of nonlinear constraints and nonlinear equality constraints would be extremely difficult to handle. b) The Feasible Direction Method of Zoutendijk |57| would result in a similar difficulty. The usual problem of zigzagging would be more acute in the presence of nonlinear equality constraints. c) The Barrier or Penalty Function Methods are more reliable, albeit slower. The main advantage of using the penalty function methods is that the constrained minimization problem is transformed to a sequence of unconstrained

minimization problems which can be solved with relative ease. These methods are quite reliable and the constraint repulsion character of the methods makes the optimization process smooth. The main drawback of the method is that the unconstrained minimization problem has to be solved a number of times and therefore the computational effort is more. Additional computer time, however, is well worth the reliability the method generally offers. Another shortcoming of these methods is the severe ill-conditioning of the unconstrained minimization problem as the value of the penalty parameter tends to its limiting value (infinity or zero). Powell |58| has suggested a penalty function that avoids the ill-conditioning in equality constrained minimization problems, and this has been modified and generalised by Osborne and Ryan |59| for both equality and inequality constraints. However, these proposals require extensive numerical experimentation and modification before they can be used for complex practical problems. The penalty function suggested by Fiacco and McCormick |60| is chosen for the present work since it has been studied quite extensively and found to be reliable. The penalty function is of the form

$$\Phi(\vec{x}, r) = f(\vec{x}) + r \sum_{j=1}^{NCC} \frac{1}{g_j(\vec{x})} + \frac{1}{\sqrt{r}} \sum_{k=1}^{M} l_k^2(\vec{x}) \qquad (4.1)$$

in which,

Φ is the penalty function,

r is the penalty parameter, and

NCC is the total number of inequality constraints in the problem.

This is an interior penalty function approach and therefore the value of r reduces for successive unconstrained minimizations.

The subroutine VA10A developed by Fletcher |61| is used for the unconstrained minimization. This program makes use of a quasi-Newton algorithm and follows the line of the Davidon-Fletcher-Powell algorithm. The Hessian matrix is approximated and updated during successive linear searches.. (In the DFP algorithm the inverse of the Hessian is approximated and updated). The program uses a single quadratic fit for linear search. Some studies by Fletcher |62| indicate that quasi-Newton methods are most efficient if the linear search is carried out with minimum number of function evaluations. It is likely that such an approach may not be as robust on ill-conditioned problems such as those that arise when using penalty functions. This aspect is discussed in Chapter 6.

4.2 GRADIENT COMPUTATION:

The unconstrained minimization procedure used in the present work is a gradient method and requires the gradient of the penalty function to be evaluated at the start of every linear search. The gradient of the penalty function is obtained by differentiating Equation 4.1 with respect to the design variables,

$$\frac{\partial \Phi}{\partial x_i} = \frac{\partial f}{\partial x_i} - r \sum_{j=1}^{NCC} \left(\frac{\partial g_j}{\partial x_i} \right) / g_j^2$$

$$+ \frac{1}{\sqrt{r}} \sum_{k=1}^{M} 2 l_k \frac{\partial l_k}{\partial x_i} \qquad (4.2)$$

$$i = 1, \ldots, N$$

Thus the gradient computation requires the evaluation of the rates of change of the objective function and the constraints with respect to the design variables. These are usually computed for such general optimization formulations by the finite difference scheme. The subroutine VA10A requires only about 3 to 4 penalty function evaluations per linear search, and the total number of linear searches is increased because the linear search is not very accurate. Thus the use of this program for the optimization problem with N design variables, combined with a forward or backward difference scheme would require about N+4 penalty function evaluations per linear search, out of which N+1 would be required for the gradient computation alone. Thus any alternative procedure which leads to a faster computation of the gradient would improve the efficiency of the optimization program substantially.

A major portion of the computational effort in the evaluation of the objective function and the constraints at a design point is spent in computing the stiffness matrices of the elements, and in the solution of the equilibrium equations.

The use of the finite difference scheme results in a major portion of this effort being repeated several times; this repetition, if avoided, can result in a considerable saving in the computer time. Alternative procedures for gradient evaluation are developed and implemented for the synthesis and the integrated synthesis approaches. The procedures are based essentially on the techniques of reanalysis of modified structures, which are discussed in the next section. The procedure for gradient computation for the synthesis approach is detailed in Section 4.4 and that for the integrated synthesis approach is given in Section 4.5.

The fully constrained design formulation is not amenable to such explicit gradient computation procedures as used for the other two formulations because it is basically an iterative design procedure. Furthermore, the number of design variables in this formulation is considerably small, and any saving of computer time obtained in the computation of gradients would not be significant. The gradient is, therefore, obtained for this formulation by the forward difference scheme.

4.3 METHODS OF REANALYSIS:

The problem of reanalysis may be stated as follows: Given a design of a structure, its stiffness matrix, and its displacements and stresses under a set of loads, what is the best method of obtaining the deformations and stresses of a modified design?

The general methods of solution adopted for this problem are classified into a) Taylor Series Expansion or the First Order Variation Methods, b) Iteration, c) Power Series Expansion, and d) Reduced Basis Methods.

The Taylor series expansion methods are more widespread than the others. Consider the set of equilibrium equations,

$$K \vec{u} = \vec{p} \tag{4.3}$$

of the starting design. When the design is modified, the stiffness matrix changes from K to $K + \Delta K$ and the deformations change from $\vec{u}$ to $\vec{u} + \Delta\vec{u}$. The equilibrium equations of the modified design are given by

$$[K + \Delta K] \{\vec{u} + \Delta\vec{u}\} = \vec{p} \tag{4.4}$$

or

$$K \vec{u} + \Delta K \vec{u} + K \Delta\vec{u} + \Delta K \Delta\vec{u} = \vec{p} \tag{4.5}$$

By virtue of Equation 4.3 and ignoring the second order term $\Delta K \Delta\vec{u}$, and rearranging, we obtain,

$$\Delta\vec{u} = - K^{-1} \Delta K \vec{u} \tag{4.6}$$

The analysis of the starting design would have K^{-1} and $\vec{u}$ in the memory of the computer, or, if the solution of Equation 4.3 is obtained using the Cholesky decomposition, the decomposed upper or lower triangular matrix and $\vec{u}$ would be available. Computation of ΔK followed by matrix multiplications would give $\Delta\vec{u}$ and thereby $\vec{u} + \Delta\vec{u}$. This procedure is applicable to small modifications in the design in which the effect of ignoring the

term $\Delta K \Delta \vec{u}$ would not be significant. For large changes in the design, when the elements in ΔK would be comparable to those in matrix K, the following equation is used:

$$\Delta \vec{u} = - [K + \Delta K]^{-1} \Delta K \, \vec{u} \tag{4.7}$$

The matrix $[K + \Delta K]^{-1}$ is computed from K^{-1} using the Sherman and Morrison formula |63|. Several variations of this basic procedure have been used by Kirsch and Rubinstein |64|, Argyris et al |65,66,67| and Kavlie and Powell |68|.

Kirsch and Rubinstein |69| have considered the problem of reanalysis by iteration. The iteration scheme is basically given by,

$$\vec{u}^{(i+1)} = \vec{u} + K^{-1} \Delta K \, \vec{u}^{(i)} \tag{4.8}$$

in which the $\vec{u}$ with a superscript denotes the displacement for the modified design and the superscript denotes the iteration count. For large changes in the design, the convergence of the iterative procedure was found to be poor, or even not possible.

Romstad et al |70| have used a power series expansion procedure to compute static and dynamic structural response of modified structures. This, the authors state, was necessitated because the first order Taylor series expansion was found to be inadequate for the optimum design with stability constraints. Kirsch and Rubinstein's iterative scheme |69| given by

Equation 4.8 can also be interpreted as a kind of power series expansion.

The reduced basis method was proposed by Fox and Miura |71| in which the analysis of a modified design is expressed as a linear combination of the exact analyses performed earlier for different designs, and the problem reduces to the solution of a smaller system of equations. The method lacks the mathematical rigour, and the accuracy of the solution obtained is not predictable. Recently, Noor and Lowder |72| have proposed a rational choice of the reduced basis vectors. The method would be competitive only for problems in which the number of degrees of freedom is very large as compared with the number of design variables.

Noor and Lowder |72| have also considered the accuracy and convergence characteristics of a reanalysis procedure combining a single term Taylor series expansion followed by iteration, and have obtained accurate results with very few iterations even for significant changes in the design variables.

In problems of reanalysis, it is very useful to consider the concept of 'Sensitivity Analysis Vectors'. Essentially the first order sensitivity analysis vector for a design variable is the rate of change in the analysis vector (e.g. displacements) with respect to a change in that design variable. This is also the result of expanding the analysis vector in the

Taylor series at a design point with respect to the design variables. For example,

$$\vec{u}\,(\vec{x} + \Delta\vec{x}) = \vec{u}(\vec{x}) + \sum_{i=1}^{N} \frac{\partial \vec{u}}{\partial x_i}\Big|_{\vec{x}} \Delta x_i + \text{error} \qquad (4.9)$$

$\frac{\partial \vec{u}}{\partial x_i}\Big|_{\vec{x}}$ is defined as the sensitivity analysis vector for displacements at point $\vec{x}$ with respect to the i-th design variable. This concept has been used for static reanalysis by Noor and Lowder |72| and for gradient computation in the present work. A similar concept is used for eigenvalue analysis by Fox and Kapoor |73| and for flutter analysis by Rao |74|.

Another procedure which is gaining popularity of late is the method of successive approximations. This procedure tries to reduce the total number of analyses performed during an optimization procedure. The objective function and the constraints of the optimization problem are expressed in terms of linear or quadratic Taylor series expansions. The minimization is performed for the approximated function and constraints. Successive approximations and minimizations lead the solution to the optimum of the actual problem. The advantage of these procedures is that very few number of analyses is required to make the approximations, and the optimization procedure itself does not require any additional analysis. Thus the number of analyses performed during an optimization procedure is considerably reduced. The linearisation concept has been used by

Romstad and Wang |10| and by Schmit and Farshi |75|. The quadratic expansion technique has been studied by Lund |76|. This approach seems to be very attractive when combined with the Sequential Unconstrained Minimization Technique.

The procedures used for the computation of gradients in the present work are given in the next two sections. In the synthesis approach the Taylor series expansion is used to obtain the sensitivity analysis vectors for displacements and then the finite difference scheme is used to obtain the rates of change of the constraints. In the integrated synthesis approach, closed form differentiation of the expressions is used where available. Finite differences are used where closed form differentiation is not possible. In both the cases, the unnecessary computations are avoided.

4.4 GRADIENT COMPUTATION FOR THE SYNTHESIS APPROACH:

Let $\vec{x}$ be the current design vector, where the gradient is required. Let K and $\vec{u}$ be the stiffness matrix and displacement vector respectively at this point. A small change Δx_j in the j-th design variable causes a change of ΔK_j in the stiffness matrix and the change in the displacement vector $\Delta \vec{u}_j$ is given by,

$$\Delta \vec{u}_j = - K^{-1} \Delta K_j \vec{u} \tag{4.10}$$

In this formulation, the design variables excluding the pretensions in the cables are all properties defining the cross section of any one member. The quantity ΔK_j can therefore be computed, for these variables, from the stiffness matrix of the element i, to which the variable j is related. In the linear elastic analysis of the structure, the stiffness matrix of the element is linearly related to its area of cross section A and moment of inertia I.

$$S_i = S(A_i, I_i) \tag{4.11}$$

$$\left[\frac{\partial S_i}{\partial x_j}\right] = S\left(\frac{\partial A_i}{\partial x_j}, \frac{\partial I_i}{\partial x_j}\right) \tag{4.12}$$

$$\Delta S_{ij} = \left[\frac{\partial S_i}{\partial x_j} \Delta x_j\right]$$

$$= S\left(\frac{\partial A_i}{\partial x_j} \Delta x_j, \frac{\partial I_i}{\partial x_j} \Delta x_j\right) \tag{4.13}$$

Thus ΔS_{ij}, the change in the stiffness matrix of element i due to a change Δx_j in the variable j can be obtained from the expressions for the stiffness matrix by simply substituting $\frac{\partial A_i}{\partial x_j}\Delta x_j$ and $\frac{\partial I_i}{\partial x_j}\Delta x_j$ for A_i and I_i respectively. In the non-linear analysis, the expression for the derivative of the stiffness matrix is slightly more involved due to the presence of the axial load parameter ϕ. The relevant expressions required in this case are given in Appendix A.

Since only the stiffness of member i is affected by modifying the variable j, ΔS_{ij} alone contributes to the matrix ΔK_{ij}. The vector $\Delta \vec{u}_j$ is computed using Equation 4.10 and the displacements for the perturbed design are obtained as $\vec{u}_j = \vec{u} + \Delta \vec{u}_j$.

If the design variable is the pretension force in the cables, only the load vector changes from $\vec{p}$ to $\vec{p} + \Delta \vec{p}$ when this variable is changed. The displacement corresponding to this change is directly obtained,

$$\vec{u}_j = \vec{u} + \Delta \vec{u}_j = \vec{u} + K^{-1} \Delta \vec{p} \tag{4.14}$$

For a change Δx_j in the design variable x_j, the constraints at the perturbed design point are evaluated as g_{kj}, k = 1, ..., NCC. The rate of change of the constraints is, therefore,

$$\frac{\partial g_k}{\partial x_j} \simeq \frac{g_{kj} - g_k}{\Delta x_j} , \qquad k = 1, \ldots, NCC \tag{4.15}$$

The cost of member i is

$$C_i = c_i \; L_i \; A_i \tag{4.16}$$

where c_i is the cost per unit volume of member i, and L_i, its length. The rate of change of the cost of member i with respect to the change in the variable j is

$$\frac{\partial C_i}{\partial x_j} = c_i \; L_i \; \frac{\partial A_i}{\partial x_j} \tag{4.17}$$

Again since only member i is related to the variable j, this expression is also the rate of change of the objective function with respect to the change in the variable j,

$$\frac{\partial f(\vec{x})}{\partial x_j} = \frac{\partial C_i}{\partial x_j} \tag{4.18}$$

Substituting the expressions 4.15 and 4.18 in Equation 4.2 and noting that no equality constraints are present in this formulation, we obtain,

$$\frac{\partial \Phi}{\partial x_j} = \frac{\partial C_i}{\partial x_j} - r \sum_{k=1}^{NCC} \frac{g_{kj} - g_k}{\Delta x_j \cdot g_k^2} \tag{4.19}$$

which is the j-th element of the gradient vector of the penalty function.

4.5 GRADIENT COMPUTATION FOR THE INTEGRATED SYNTHESIS APPROACH:

The design vector $\vec{x}$ in this formulation consists of three types of variables: nodal displacements, geometrical properties defining the cross sections of members and the pretension forces in the cables. These are treated separately for the purpose of development of the gradient expressions.

Let x_j be the nodal displacement at joint JT. A change in this displacement causes a change in the equilibrium state of the related joints. The equilibrium constraint is given by Equation 2.12 as,

$$l_k = \frac{1}{FMAX_k} \left[\sum_{r=1}^{M} K_{kr} u_r - p_k \right] , \quad k = 1, \ldots, M$$

The member end forces and consequently the stress resultants change only for member i. Therefore, the stress constraints at the perturbed design g_{kj} need only to be calculated for member i, and the rate of change of these constraints is then given by Equation 4.21. For the constraints related to all the other members, the rate of change due to a change in x_j is zero. The rate of change of the objective function in this case is given by Equation 4.18.

Finally, let x_j be the pretension in the cable i. A change Δx_j in the pretension would cause a change $\Delta \vec{p}$ in the load vector and the rate of change of the equilibrium equations would then be

$$\frac{\partial l_k}{\partial x_j} = - \frac{1}{FMAX_k} \frac{\Delta p_k}{\Delta x_j} \qquad k = 1, \ldots, M \qquad (4.24)$$

This change also affects the two stress constraints of the cable. The rate of change of the two constraints can be written in closed form as,

$$\frac{\partial g_k}{\partial x_j} = - \frac{1}{A_i \times 0.6\, \sigma_{yc}} \qquad k \in NCC \qquad (4.25)$$

and

$$\frac{\partial g_k}{\partial x_j} = - \frac{1}{A_i \times 0.8\, \sigma_{yc}} \qquad k \in NCC \qquad (4.26)$$

Equation 4.25 relates to the constraint on the pretension force and Equation 4.26 to the constraint on the final force in the cable. The other stress constraints and the objective function are unaffected by a change in this variable.

The gradient of the penalty function with respect to the variables is obtained by substituting the appropriate rates of change as detailed above in the expression for the gradient given by Equation 4.2.

CHAPTER FIVE

ILLUSTRATIVE EXAMPLES

The formulations discussed earlier are applied to the minimum cost design of three different cable stayed structures. Furthermore, the synthesis approach is used to study the effect of variation of some of the predetermined parameters on the optimum design. The results of these studies are presented in this chapter. All the results given herein were obtained on the IBM 370/155 system. The execution times indicated for various problems are the C.P.U. times on this system.

5.1 EXAMPLE 1 – TIED CANTILEVER:

The first structure considered is a cantilever box beam supported at the tip by a high tensile cable, as shown in Fig. 5.1. The relevant details of the structure are as follow:

Member 1 is a box beam 15 cm wide and 30 cm deep, made up of mild steel of relative cost 1000 units/m^3, and member 2 is a high tensile cable, under initial tension, of relative cost 5000 units/m^3. The allowable stresses in the members are:

Box beam:	Maximum stress in combined bending and axial force	= 1250 Kg/cm^2
	Maximum average shear stress in the web	= 945 kg/cm^2

Cable: Yield stress of the cable, σ_{yc} = 15000 kg/cm^2

Maximum pretension stress = 0.6 σ_{yc} = 9000 kg/cm^2

Maximum stress under working load = 0.8 σ_{yc} = 12000 kg/cm^2

The vertical deflection at joint 3 is constrained so as not to exceed $\pm$ 1.5 cm.

The synthesis approach for this example has 4 design variables, 4 stress constraints, 1 deflection constraint and 4 side constraints. The integrated synthesis approach leads to a problem with 7 design variables, 3 equilibrium constraints, 4 stress constraints, and 7 side constraints. The fully constrained design formulation, which leads to an unconstrained minimization problem, has only one design variable for this example.

Two types of loads are considered on this structure leading to two problems. In Problem 1, the structure is loaded by a single downward load at joint 3 of magnitude 40 Tonnes, and in Problem 2, the beam is loaded by a downward uniformly distributed load of 8T/m. For each of the problems, two cases are studied; case a in which the minimum thicknesses of the flange and web of the beam are restricted to 6 mm and case b where no such restriction on the thicknesses is imposed. Both linear and nonlinear analyses are considered for obtaining the optimum design via the synthesis and the integrated synthesis

approaches. However, only the linear behaviour is considered for the fully constrained design formulation, for reasons to be explained later.

The results of the three formulations for Problem 1 are presented in Tables 5.1, 5.2 and 5.3, and those for Problem 2, in Tables 5.4, 5.5 and 5.6. It may be observed that with the exception of Problem 1 case b, all the three formulations lead to the same design in all the problems (within a certain numerical accuracy) and also that the final design is a fully constrained design. It may also be observed that the optimum design in the case of Problem 1 case b is such that the area of the box beam is nearly constant although the thicknesses of the web and flange are quite different in all the cases. This is discussed in greater detail in Section 6.2.

5.2 EXAMPLE 2 - SYMMETRIC CABLE STAYED GIRDER:

The second structure is a part of the symmetric cable stayed girder as shown in Figure 5.2. Only the portion of the girder to the left or right of the suspended span is considered for optimization. This portion is shown in Figure 5.3.

The girder is a box beam 200 cm wide and 300 cm deep, and is supported by 3 high tensile cables under initial tension. The cables meet at the top of the tower, and the tower is also

assumed as a box section 200 cm wide and 300 cm deep. The allowable stresses in the beams and cables are the same as those used for the previous example.

The synthesis approach to the optimum design of this structure leads to a nonlinear programming problem with 16 design variables, 16 stress constraints and 16 side constraints. The integrated synthesis approach has 31 design variables, 15 equilibrium constraints, 16 stress constraints and 31 side constraints. The fully constrained design formulation has only three design variables.

Once again, two types of loads, (i) only joints loaded and (ii) members and joints loaded are studied for this structure. Problem 3 considers the total loads on the structure to be lumped at the nodes of the structure; thus joints 2,3,5 and 6 are subjected to downward loads of 385T, 60T, 340T and 330T respectively. Problem 4 considers the loads on the girder to be uniformly distributed at 10T/m. Besides, joints 3 and 6 have downward loads of 60T and 300T respectively.

The integrated synthesis approach failed to give meaningful results for this problem because of premature termination of the program. The experiences with this formulation are discussed in the next chapter. Therefore, the results are presented only for the synthesis approach and for the fully constrained design formulation.

The results, using the synthesis approach, of the linear and nonlinear formulations for Problem 3, with costs of the materials as 1 and 3 units/m^3 for beam and cable, are presented in Table 5.7. The results show that the minimum cost design is a fully constrained design. To ascertain this fact, a few variations are studied. If the design is a fully constrained design, the variation in the costs of the materials should not affect the total quantity of the materials used. Indeed it is found to be so, when the relative cost of the cable is changed from 3 to 5 units/m^3. This result is presented in Table 5.8.

The above cases consider the lower bound on the thickness of the flanges and the webs to be 16 mm. This bound is reduced to 10 mm and the problem is solved again for the two cost ratios mentioned above. These results are also presented in Table 5.8. All these results confirm that the optimum design is a fully constrained design.

It may be observed that for this problem, the nonlinear behaviour has very little effect on the optimum total cost. The optimum total cost of materials is reduced slightly if the nonlinear behaviour is considered.

The results, using the synthesis approach, under linear and nonlinear behaviour, of Problem 4 are presented in Table 5.9. The minimum thickness limitation of 16 mm and the unit costs of 1 and 3 units per m^3 for beam and cable respectively are

considered. The reduction of the cost of the structure with successive values of the penalty parameter 'r' are shown in Figure 5.4. The effect of nonlinearity in this problem is once again observed to be very small. However, the consideration of nonlinear effects results in an increase in the optimum total cost of the materials.

The results of Problems 3 and 4 using the fully constrained design approach are presented in Table 5.10. The minimum thickness limitation is 16 mm and the unit costs are 1 and 3 units per m^3 for the beam and the cable respectively. For Problem 4, the reduction in the cost of the structure with respect to the cumulative number of linear searches performed for unconstrained minimization is shown in Figure 5.5. The variation of the design variables during the search procedure is shown in Figure 5.6. It is seen from Figure 5.5 that the fully constrained design at the starting point is only about 1 percent costlier than the optimum design. This indicates that the region over which the optimization is to be performed is rather 'shallow'.

5.3 EXAMPLE 3 - NONSYMMETRIC CABLE STAYED GIRDER:

The third structure considered is shown in Figure 5.7. It is a slight variant of the one studied by Tang |25| and Morris |27|. The structure consists of 4 beam elements, 1 tower and 2 cables under initial tension. The beams and the tower are box girders 250 cm wide and 350 cm deep. The structure is

loaded by a uniformly distributed load of 25T/m and the top of the tower is subjected to a downward load of 85T. The lower bound on the thickness of flange and web of the beam elements is 12 mm. The allowable stresses are the same as the ones used in the earlier problems. This problem is henceforth referred to as Problem 5.

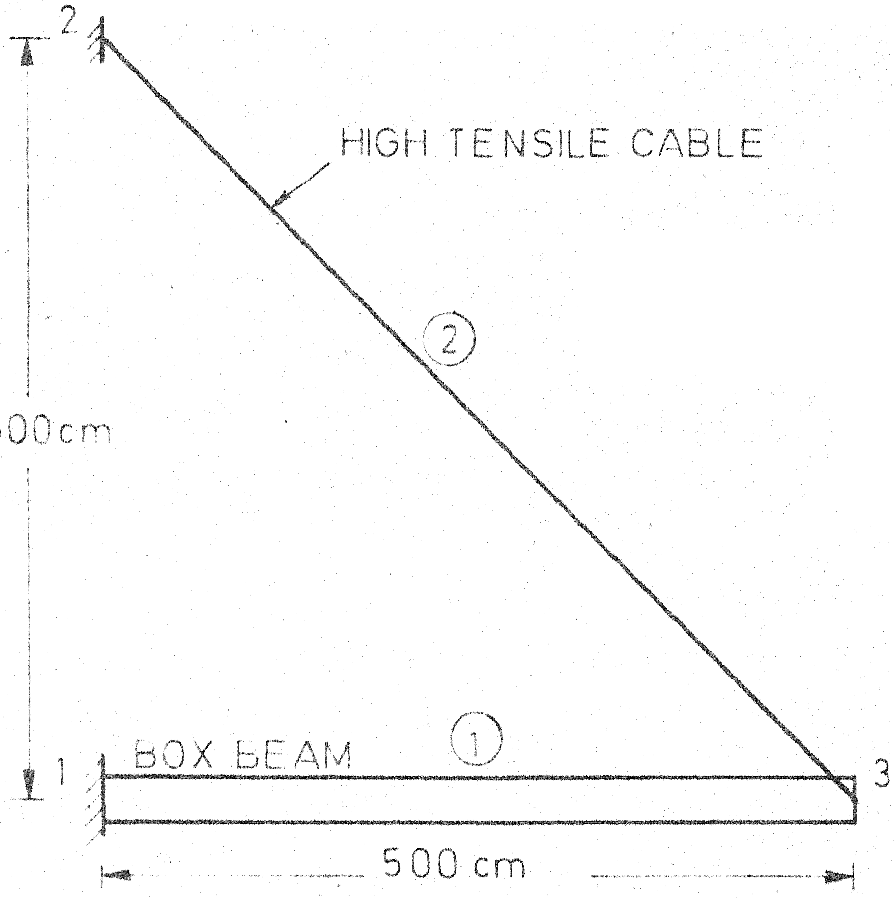
The synthesis approach as applied to this problem has 14 design variables, 14 stress constraints and 14 side constraints. The results of optimization for the linear and nonlinear behaviour of the structure are presented in Table 5.11. It may be observed again that the effect of considering the nonlinear behaviour on the optimum design is very small, and that the optimum design has a slightly increased cost. The reduction in the cost with respect to successive values of the penalty parameter is shown in Figure 5.8.

The fully constrained design formulation as applied to this problem has only two design variables. The results of the problem using this formulation are presented in Table 5.12. The reduction in the cost of the structure with respect to the cumulative number of linear searches performed is shown in Figure 5.9, and the variation of the design variables during the search procedure is shown in Figure 5.10. It is seen from Figure 5.9 that the fully constrained design at the starting point is 4.5 percent costlier than the optimum design. It

may be observed that the time taken for arriving at the optimum is only about a tenth of the time taken by the synthesis approach.

In order to study the effect of the depth of the section on the extent of nonlinearity that would be present in the optimum design, a parametric study of the variation of the optimum design with respect to the depth of the girder is made. The girder is assumed to be a box 600 cm wide, and the depth is varied from 150 cm to 350 cm at intervals of 50 cm. This problem is referred to as Problem 6. The results of these studies for the linear behaviour of the structure are presented in Table 5.13 and for the nonlinear behaviour, in Table 5.14. The variation of the optimum cost with respect to the depth of the girder is shown in Figure 5.11. It may be observed that the effect of nonlinearity on the optimum design is only marginal in all cases, and that the pretensions in the cables undergo relatively more significant changes for smaller depths.

The structure is optimised for a depth of girder of 150 cm and width of 600 cm, with the yield stress σ_{yc} of the cable reduced to 10000 kg/cm^2 from 15000 kg/cm^2 considered earlier. The results for both the linear and nonlinear analyses are presented in Table 5.15. It is seen that the total quantity of mild steel remains practically unaltered and that the quantity of high tensile cable increases almost in proportion to the reduction in the allowable stresses.



LOADING:

PROBLEM 1: DOWNWARD LOAD 40 TONNES AT JOINT 3

PROBLEM 2: DOWNWARD U.D.L. 8 TONNES /m ON MEMBER 1

FIG. 5·1 CANTILEVER BOX BEAM SUPPORTED AT THE TIP
PROBLEMS 1 AND 2

Table 5.1: Results of Problem 1 by the Synthesis Approach.

Member	Qty.	Linear Variable	Linear Stress Constraint	Nonlinear Variable	Nonlinear Stress Constraint
		Case a minimum thickness = 0.6 cm			
1	t_w, cm	0.6000	0.0001	0.6000	0.0000
	t_f, cm	0.6000	0.9863	0.6002	0.9863
2	A, cm^2	4.8573	0.0001	4.7971	0.0000
	P, Tonnes	43.714	0.0409	43.179	0.0288
Def.Constr.		0.1663		0.0804	
Cost		44.1733		43.9639	
No.of Functions		230		253	
No.of Gradients		61		50	
		Case b minimum thickness = 0.0 cm			
1	t_w, cm	0.1881	0.0000	0.5903	0.0002
	t_f, cm	0.7980	0.9909	0.0234	0.9984
2	A, cm^2	6.2117	0.0000	6.0397	0.0002
	P, Tonnes	55.903	0.2430	54.355	0.2206
Def.Constr.		0.9572		0.9171	
Cost		39.5748		39.4142	
No.of Functions		257		279	
No.of Gradients		50		63	

Note: The non-dimensional stress constraints in the tables are in the order given in Table 2.1 for the type of member.

Table 5.2: Results of Problem 1 by the Integrated Synthesis Approach.

Member	Qty.	Case a				Case b			
		Linear		Nonlinear		Linear		Nonlinear	
		Variable	Const.	Variable	Const.	Variable	Const.	Variable	Const.
	u_1, cm	-0.1742	0.0010	-0.1743	0.0001	-0.2739	-0.0044	-0.2491	0.0036
	u_2, cm	-1.3711	0.0004	-1.4363	-0.0027	-0.2696	-0.0001	-0.6216	0.0057
	u_3	-0.0041	0.0038	-0.0044	-0.0005	-0.0008	0.0157	-0.0020	0.0579
1	t_w, cm	0.6002	0.0002	0.6000	0.0006	0.5114	0.0006	0.6217	0.0000
	t_f, cm	0.6004	0.9863	0.6000	0.9863	0.1347	0.9986	0.0288	0.9978
2	A, cm^2	4.8586	0.0007	4.7991	0.0000	6.2897	0.0006	5.7990	0.0051
	P, Tonnes	43.699	0.0411	43.191	0.0292	56.570	0.2512	51.925	0.1886
Cost		44.1904		43.9684		39.6010		39.5866	
No. of Functions		411		456		473		482	
No. of Gradients		109		126		121		110	

Note: The first three constraints are equilibrium constraints in the direction of the displacement variables. The rest are stress constraints for the corresponding members.

Table 5.3: Results of Problem 1 by the Fully Constrained Design Approach.

Member	Qty.	Case a	Case b
1	t_w, cm	0.6000	0.0024
	t_f	0.6001	1.1795
2.	A, cm^2	4.8569	6.1612
	P, Tonnes	43.712	55.450
Starting Design	P_2, Tonnes	40.0	40.0
Cost		44.1729	39.5479
No. of Designs		119	233

Note: The number of designs performed is used as an indication of time rather than the number of function and gradient evaluations.

Table 5.4: Results of Problem 2 by the Synthesis Approach.

Member	Qty.	Linear Variable	Linear Const.	Nonlinear Variable	Nonlinear Const.
		Case a			
1	t_w, cm	0.6011	0.0000	0.6002	0.0002
	t_f, cm	3.0073	0.3125	3.0087	0.3114
2	A, cm^2	3.8460	0.0001	3.8722	0.0002
	P, Tonnes	34.611	0.4923	34.843	0.4957
Def. Const.			0.1874		0.1636
Cost		76.7392		76.8252	
No. of Functions		269		323	
No. of Gradients		70		80	
		Case b			
1	t_w, cm	0.4134	0.0001	0.4133	0.0000
	t_f, cm	3.1572	0.0003	3.1567	0.0000
2	A, cm^2	3.8327	0.0001	3.8591	0.0000
	P, Tonnes	34.492	0.4906	34.731	0.4941
Def. Const.			0.2001		0.1761
Cost		73.3098		73.3929	
No. of Functions		334		476	
No. of Gradients		77		97	

Table 5.5: Results of Problem 2 by the integrated synthesis approach.

Member	Qty.	LINEAR VAR	LINEAR CONS	NONLINEAR VAR	NONLINEAR CONS
		Case a			
	u_1, cm	-0.0312	-0.0009	PREMATURE TERMINATION	
	u_2, cm	1.3522	-0.0004		
	u_3	0.0084	0.0013		
1	t_w, cm	0.6004	0.0002		
1	t_f, cm	3.0086	0.3117		
2	A, cm^2	3.8472	0.0005		
2	P, Tonnes	34.608	0.4925		
Cost		76.7424			
No. of Functions		436			
No. of Gradients		112			
		Case b			
	u_1, cm	-0.0329	-0.0041	-0.0329	0.0085
	u_2, cm	1.3409	-0.0224	1.3558	0.0090
	u_3	0.0083	0.0133	0.0084	-0.0008
1	t_w, cm	0.4147	0.0012	0.4196	0.0012
1	t_f, cm	3.1579	0.0035	3.1586	0.0156
2	A, cm^2	3.8396	0.0024	3.8798	0.0050
2	P, Tonnes	34.474	0.4922	34.744	0.4968
Cost		73.3825		73.6865	
No. of Functions		387		412	
No. of Gradients		89		95	

Table 5.6: Results of Problem 2 by the Fully Constrained Design Approach.

Member	Qty.	Case a	Case b
1	t_w, cm	0.6000	0.4133
	t_f, cm	3.0096	3.1564
2	A, cm^2	3.8421	3.8337
	P, Tonnes	34.579	34.503
Starting Design	P_2, Tonnes	40.0	40.0
Cost		76.7279	73.2982
No. of Designs		87	97

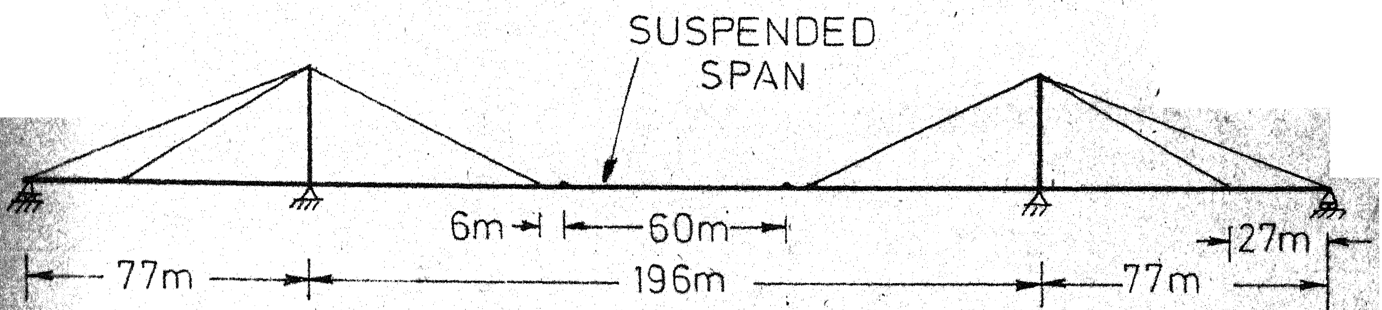
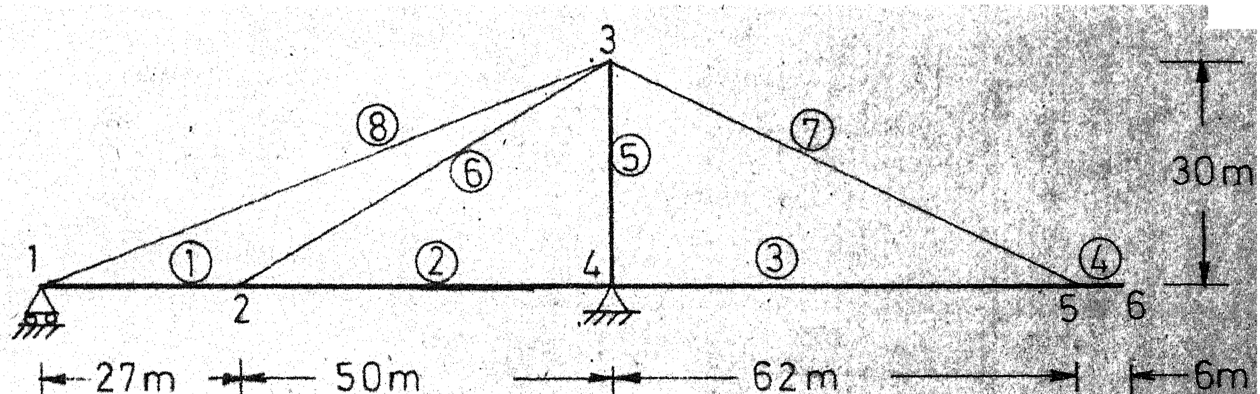


FIG. 5.2 SYMMETRIC CABLE STAYED GIRDER BRIDGE



LOADING:

PROBLEM 3: DOWNWARD LOADS OF 385 T, 60T, 340T AND 330T ON JOINTS 2,3,5 AND 6 RESPECTIVELY

PROBLEM 4: DOWNWARD UDL 10 T/m ON MEMBERS 1 TO 4. DOWNWARD LOADS OF 60 T AND 300 T ON JOINTS 3 AND 6 RESPECTIVELY

FIG. 5.3 DETAIL OF THE STRUCTURE CONSIDERED PROBLEMS 3 AND 4

Table 5.7: Results of Problem 3 by the Synthesis Approach

Cost Ratio 1:3; Minimum Thickness 1.6 cm.

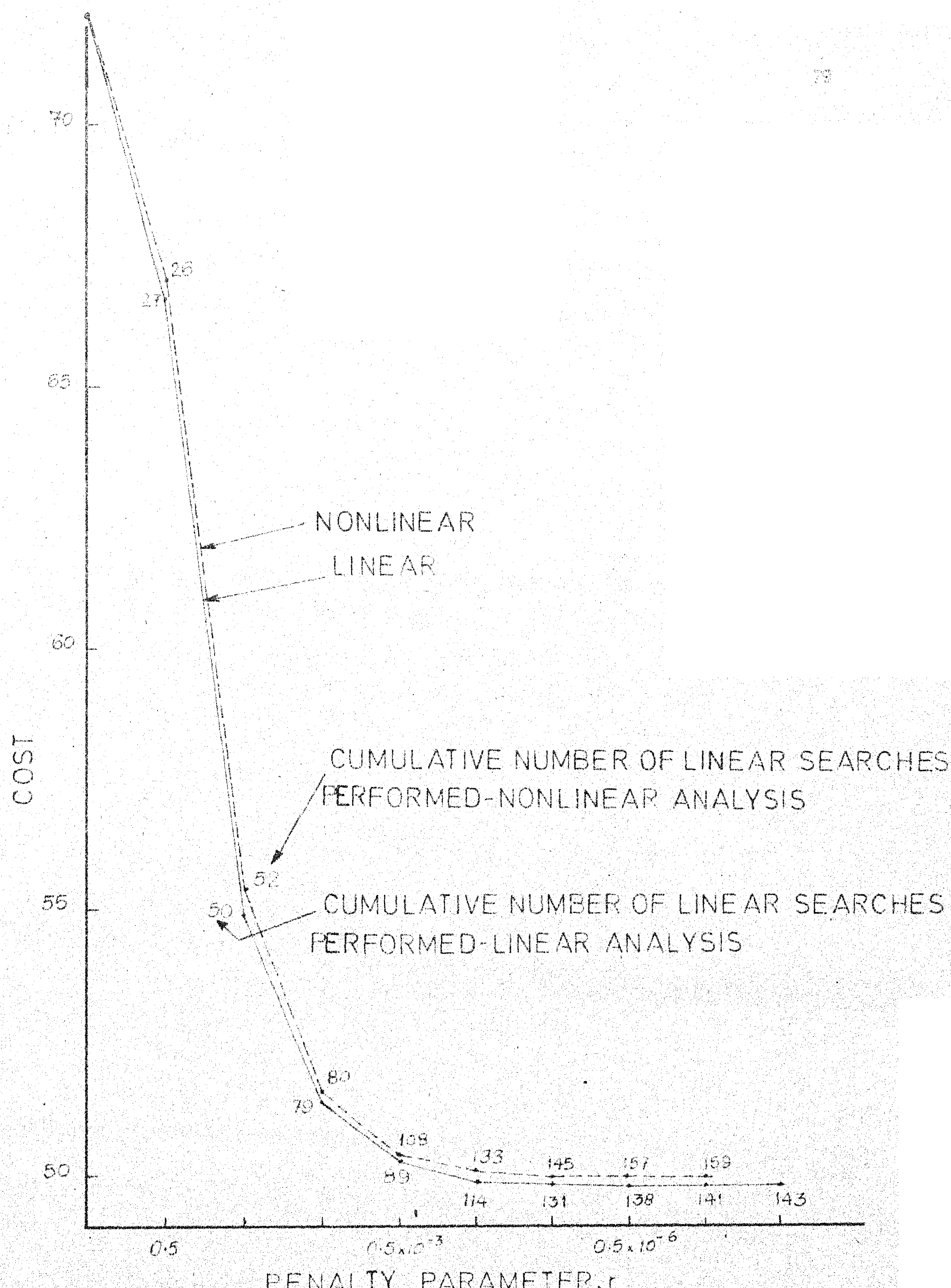
Member	Qty	Initial Design Variable	Initial Design Stress Constraint	Linear Variable	Linear Stress Constraint	Nonlinear Variable	Nonlinear Stress Constraint
1	t_w, cm	2.0	0.5371	1.6016	0.3371	1.6023	0.3360
	t_f	4.0	0.9684	1.6064	0.9791	1.6061	0.9790
2	t_w	2.0	0.3284	1.6007	0.0000	1.6027	0.0000
	t_f	4.0	0.9847	1.6037	1.0000	1.6055	1.0000
3	t_w	2.0	0.0561	1.6005	0.0001	1.6031	0.0007
	t_f	4.0	0.9697	3.9757	0.9783	3.9809	0.9768
4	t_w	2.0	0.4720	1.6119	0.0022	1.6525	0.0231
	t_f	4.0	0.7090	1.8405	0.6389	1.8716	0.6477
5	t_w	2.0	0.5102	1.6007	0.1384	1.6016	0.1874
	t_f	4.0	0.9927	1.6032	0.9909	1.6062	0.9941
6	A, cm^2	150.0	0.4444	77.180	0.0038	74.638	0.0061
	P, Tonnes	750.0	0.5268	691.98	0.1522	667.63	0.1232
7	A	280.0	0.4444	131.98	0.0139	132.40	0.0017
	P	1400.0	0.5187	1171.31	0.0002	1189.62	0.0014
8	A	125.0	0.4444	75.608	0.0008	73.266	0.0005
	P	625.0	0.4870	679.97	0.1006	659.08	0.0721
Volume of Beam (m^3)		49.000		33.980		34.040	
Volume of Cable (m^3)		3.836		1.984		1.952	
Cost		60.5084		39.9312		39.8979	
No. of Functions				450		499	
No. of Gradients				126		136	
Time, Secs.				254		336	

Table 5.8: Results of Problem 3 by the Synthesis Approach. (Influence of Cost ratio and minimum thickness).

Member	Qty.	Cost Ratio 1:3 Min.Thk.1.0 cm Variable	Cost Ratio 1:3 Min.Thk.1.0 cm Stress Const.	Cost Ratio 1:5 Min.Thk.1.0 cm Variable	Cost Ratio 1:5 Min.Thk.1.0 cm Stress Const.	Cost Ratio 1:5 Min.Thk.1.6 cm Variable	Cost Ratio 1:5 Min.Thk.1.6 cm Stress Const.
1	t_w, cm	1.0016	0.0792	1.0007	0.0011	1.6079	0.3183
	t_f	1.0032	0.9859	1.0051	0.9715	1.6070	0.9776
2	t_w	1.0468	0.0000	1.0394	0.0001	1.6077	0.0001
	t_f	1.7383	1.0000	2.1276	0.9999	1.6680	1.0000
3	t_w	1.0034	0.0002	1.0007	0.0002	1.6021	0.0013
	t_f	4.5479	0.9499	4.5368	0.9562	3.9853	0.9768
4	t_w	1.0074	0.0108	1.0045	0.0048	1.6311	0.0052
	t_f	2.1652	0.4222	2.1507	0.4205	1.8385	0.6431
5	t_w	1.0213	0.0008	1.1189	0.0008	1.6060	0.2101
	t_f	1.3918	0.9999	1.2349	1.0000	1.6061	0.9956
6	A, cm^2	84.505	0.0005	78.183	0.0010	74.421	0.0019
	P, Tonnes	760.18	0.2468	702.95	0.1690	668.52	0.1176
7	A	153.88	0.0001	147.36	0.0007	135.44	0.0009
	P	1384.73	0.1316	1325.36	0.0978	1217.91	0.0238
8	A	90.447	0.0012	78.542	0.0007	72.859	0.0016
	P	813.01	0.2203	706.35	0.1260	654.65	0.0713
Volume of Beam (m3)		28.726		29.427		34.190	
Volume of Cable (m^3)		2.300		2.120		1.969	
Cost		35.6256		40.0267		44.0344	
No.of Functions		472		614		426	
No.of Gradients		120		168		114	

Table 5.9: Results of Problem 4 by the Synthesis Approach. Cost Ratio 1:3; Minimum Thickness 1.6 cm.

Member	Qty.	Initial Design Variable	Initial Design Stress Const.	Linear Variable	Linear Stress Const.	Nonlinear Variable	Nonlinear Stress Const.
1	t_w, cm	2.0	0.3566	1.6006	0.0004	1.6005	0.0006
	t_f	4.0	0.8247	1.6026	0.8034	1.6116	0.8031
2	t_w	2.0	0.0572	1.6002	0.0002	1.6007	0.0002
	t_f	6.0	0.7501	3.9510	0.7060	3.9545	0.7062
3	t_w	2.0	0.0814	1.6005	0.0002	1.6005	0.0001
	t_f	7.0	0.6989	4.7131	0.6483	4.7133	0.6483
4	t_w	2.0	0.4720	1.6037	0.0025	1.6071	0.0065
	t_f	4.0	0.6825	1.8442	0.6042	1.8534	0.6052
5	t_w	2.0	0.4663	1.6009	0.0004	1.6009	0.0014
	t_f	4.0	0.9842	1.6011	0.9803	1.6007	0.9804
6	A, cm^2	150.0	0.4444	125.76	0.0001	127.49	0.0001
	P, Tonnes	750.0	0.5515	1131.80	0.4697	1147.29	0.4766
7	A	280.0	0.4444	206.55	0.0000	210.95	0.0001
	P	1400.0	0.5636	1858.89	0.3878	1898.42	0.4006
8	A	125.0	0.4444	113.41	0.0000	114.68	0.0002
	P	625.0	0.5382	1020.62	0.4500	1031.97	0.4566
Volume of Beam (m^3)		60.440		40.491		40.513	
Volume of Cable (m^3)		3.836		3.093		3.144	
Cost		71.9484		49.7710		49.9453	
No. of Functions				524		570	
No. of Gradients				143		159	
Time, Secs.				289		414	



FIG·5·4 PROGRESS OF OPTIMIZATION-SYNTHESIS APPROACH FOR PROBLEM 4

Table 5.10: Results of Problems 3 and 4 by the Fully Constrained Design Approach.

Member	Qty.	Problem 3	Problem 4
1	t_w, cm	1.6000	1.6000
	t_f	1.6000	1.6435
2	t_w	1.6000	1.6000
	t_f	1.6417	3.9839
3	t_w	1.6000	1.6000
	t_f	4.0012	4.7212
4	t_w	1.6000	1.6000
	t_f	1.8402	1.8398
5	t_w	1.6000	1.6000
	t_f	1.6000	1.6000
6	A, cm^2	70.731	123.28
	P, Tonnes	636.58	1109.52
7	A	153.55	208.93
	P	1381.98	1880.39
8	A	73.405	110.46
	P	660.65	994.12
Starting Design	P_6, Tonnes	750.00	1000.0
	P_7	1400.00	2000.0
	P_8	625.00	1000.0
Cost		40.2427	49.8249
No. of Designs		108	118
Time, Secs.		32	36.3

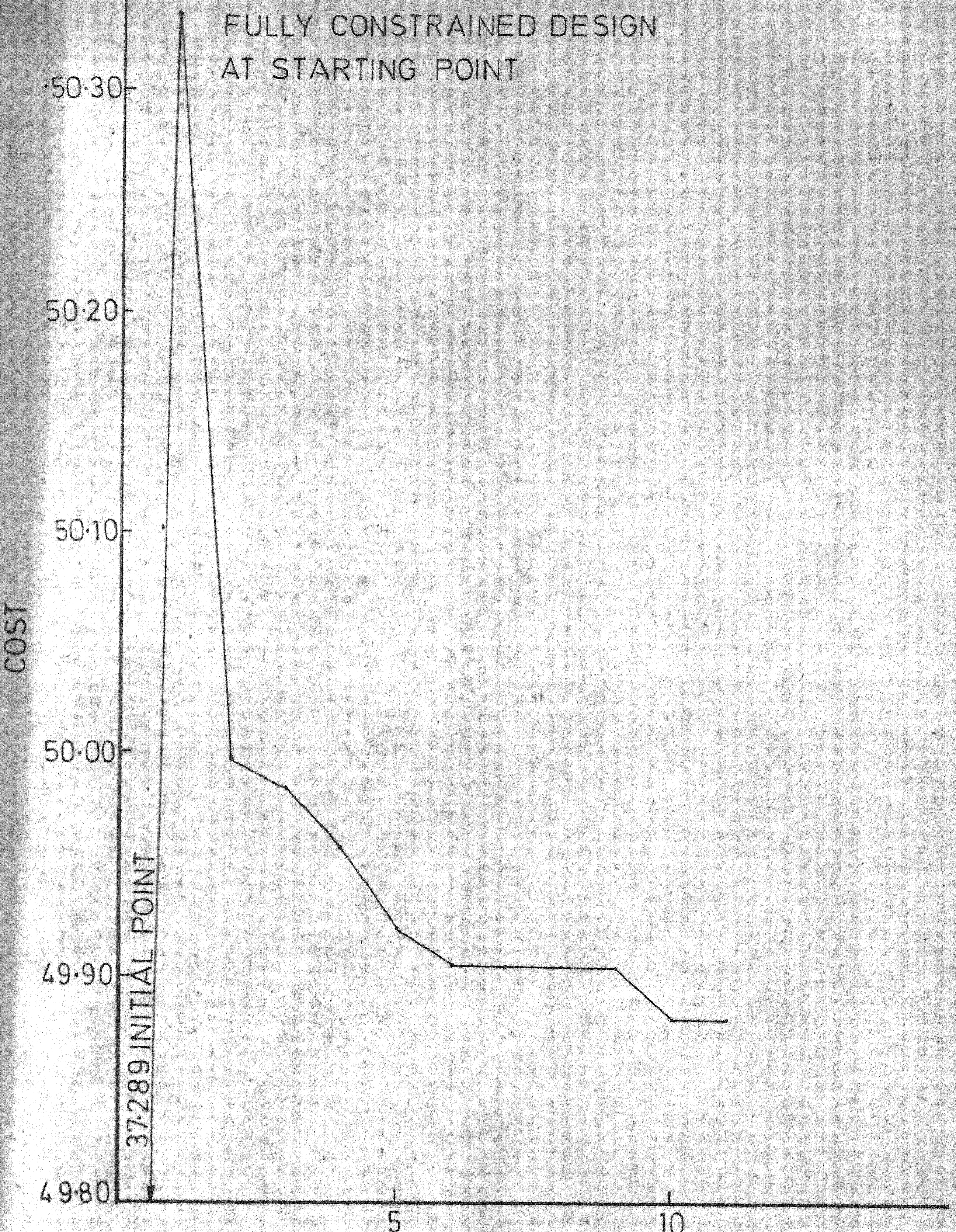


FIG.5·5 PROGRESS OF OPTIMIZATION- FULLY CONSTRAINED DESIGN APPROACH FOR PROBLEM 4

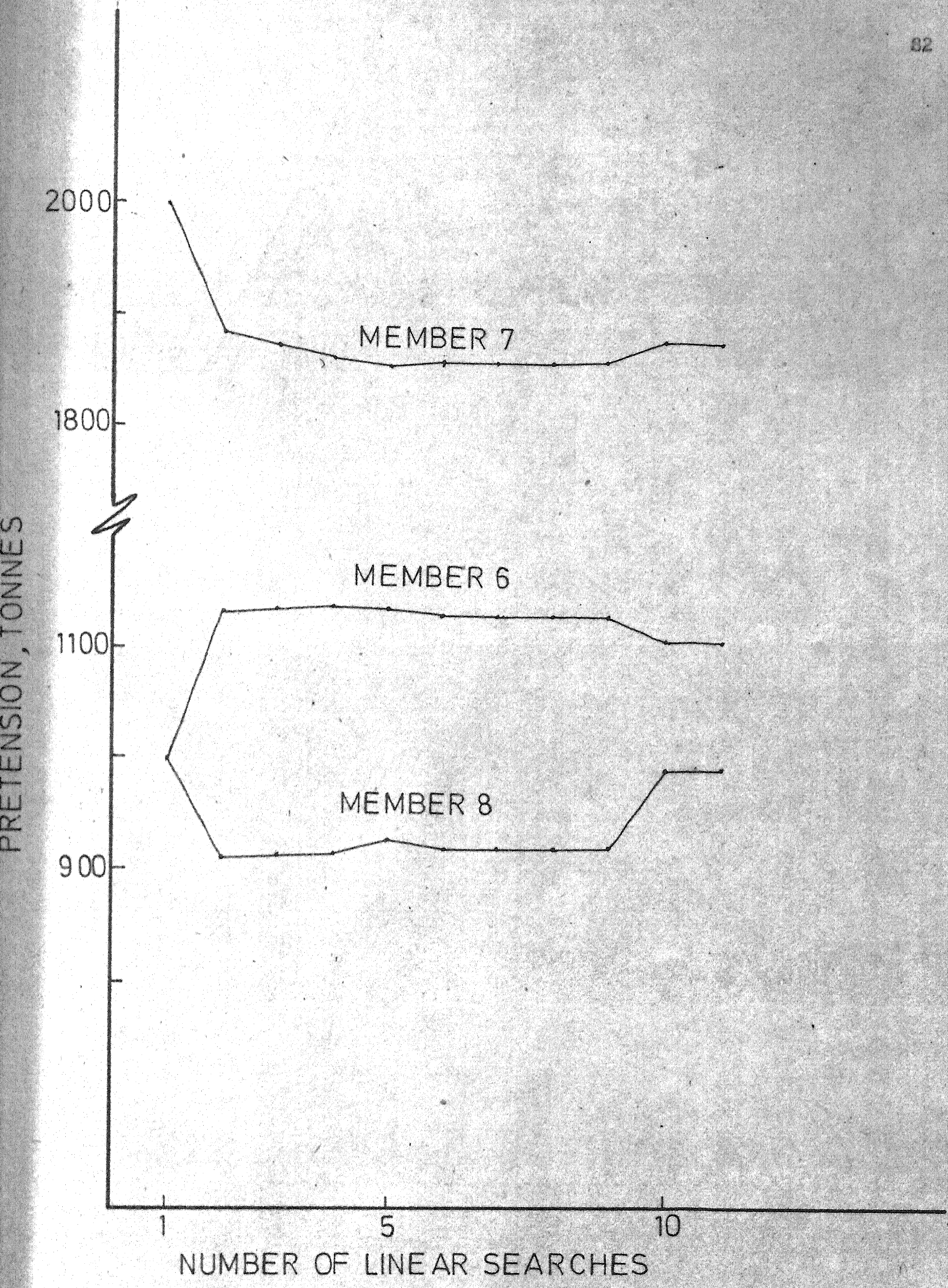
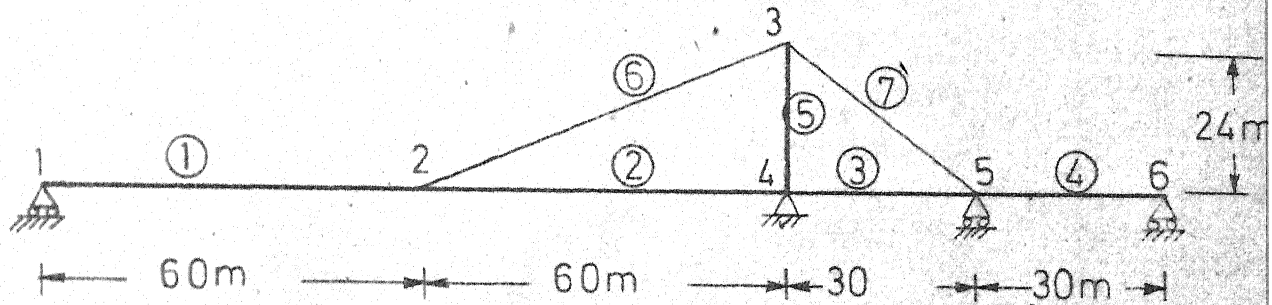


FIG.5.6 PATH OF THE DESIGN VARIABLES-FULLY CONSTRAINED DESIGN APPROACH FOR PROBLEM 4



LOADING: DOWNWARD UDL 25 T/m ON MEMBERS 1 TO 4
DOWNWARD LOAD OF 85 T ON JOINT 3

PROBLEM 5: MEMBERS 1 TO 5 ARE 250 cm WIDE AND 350 cm DEEP

PROBLEM 6: MEMBERS 1 TO 5 ARE 600 cm WIDE. DEPTH VARIES

FIG. 5.7 NONSYMMETRIC CABLE STAYED GIRDER
PROBLEMS 5 AND 6

Table 5.11: Results of Problem 5 by the Synthesis Approach.

Member	Qty.	Initial Design Variable	Initial Design Stress Const.	Linear Variable	Linear Stress Const.	Nonlinear Variable	Nonlinear Stress Const.
1	t_w, cm	2.0	0.3160	1.3266	0.0002	1.3270	0.0000
	t_f	10.0	0.3301	6.4713	0.0002	6.4595	0.0000
2	t_w	2.0	0.1596	1.2009	0.0000	1.2003	0.0000
	t_f	15.0	0.4218	12.4190	0.0559	12.4487	0.0554
3	t_w	2.0	0.3126	1.2021	0.0001	1.2005	0.0000
	t_f	15.0	0.6356	12.4160	0.3039	12.4422	0.3019
4	t_w	2.0	0.5968	1.2006	0.0003	1.2002	0.0001
	t_f	4.0	0.6617	1.5278	0.4320	1.5269	0.4318
5	t_w	4.0	0.0763	1.2689	0.0006	2.9974	0.0000
	t_f	6.0	0.9702	6.1782	0.9999	3.7635	0.9998
6	A, cm^2	1000.0	0.5000	521.40	0.0001	521.71	0.0000
	P, Tonnes	4500.0	0.6295	4692.14	0.2997	4695.26	0.2999
7	A	1000.0	0.3444	541.47	0.0003	541.44	0.0000
	P	5900.0	0.5510	4871.81	0.1982	4872.86	0.1978
Volume of Beam (m^3)		142.620		102.793		102.884	
Volume of Cable (m^3)		10.304		5.450		5.451	
Cost		173.5318		119.1422		119.2408	
No. of Functions				523		525	
No. of Gradients				134		147	
Time, Secs.				226		289	

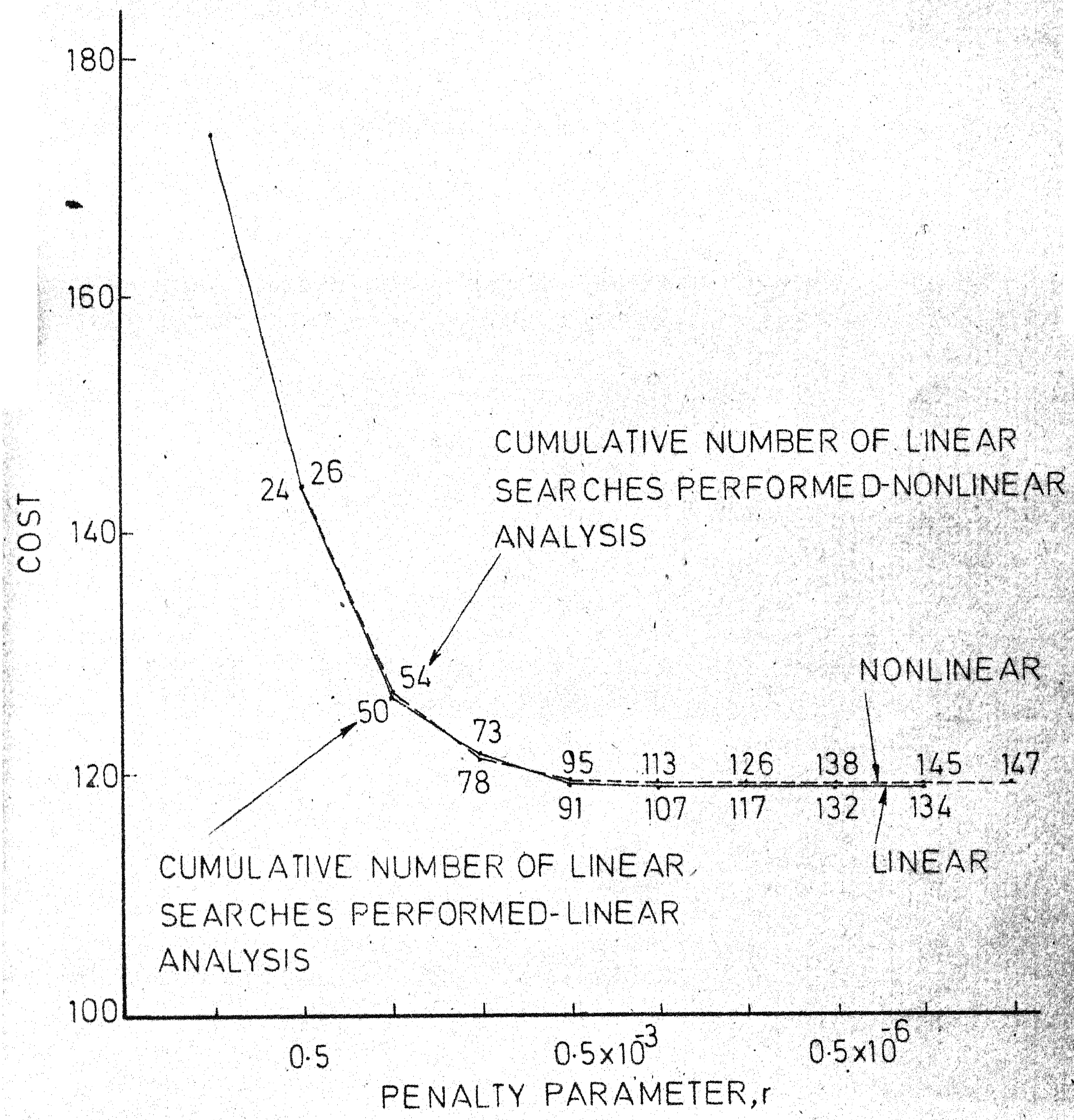


FIG. 5·8 PROGRESS OF OPTIMIZATION-SYNTHESIS APPROACH FOR PROBLEM 5

Table 5.12: Results of Problem 5 by the Fully Constrained Design Approach.

Member	Qty.	Value
1	t_w, cm	1.3281
	t_f	6.4430
2	t_w	1.2000
	t_f	12.4978
3	t_w	1.2000
	t_f	12.3236
4	t_w	1.2000
	t_f	1.5311
5	t_w	1.2000
	t_f	6.3405
6	A, cm^2	522.22
	P, Tonnes	4699.95
7	A	544.75
	P	4902.79
Starting Design	P_6	4500.00
	P_7	5900.00
Cost		119.2891
No. of Designs		83
Time, Secs.		21.22

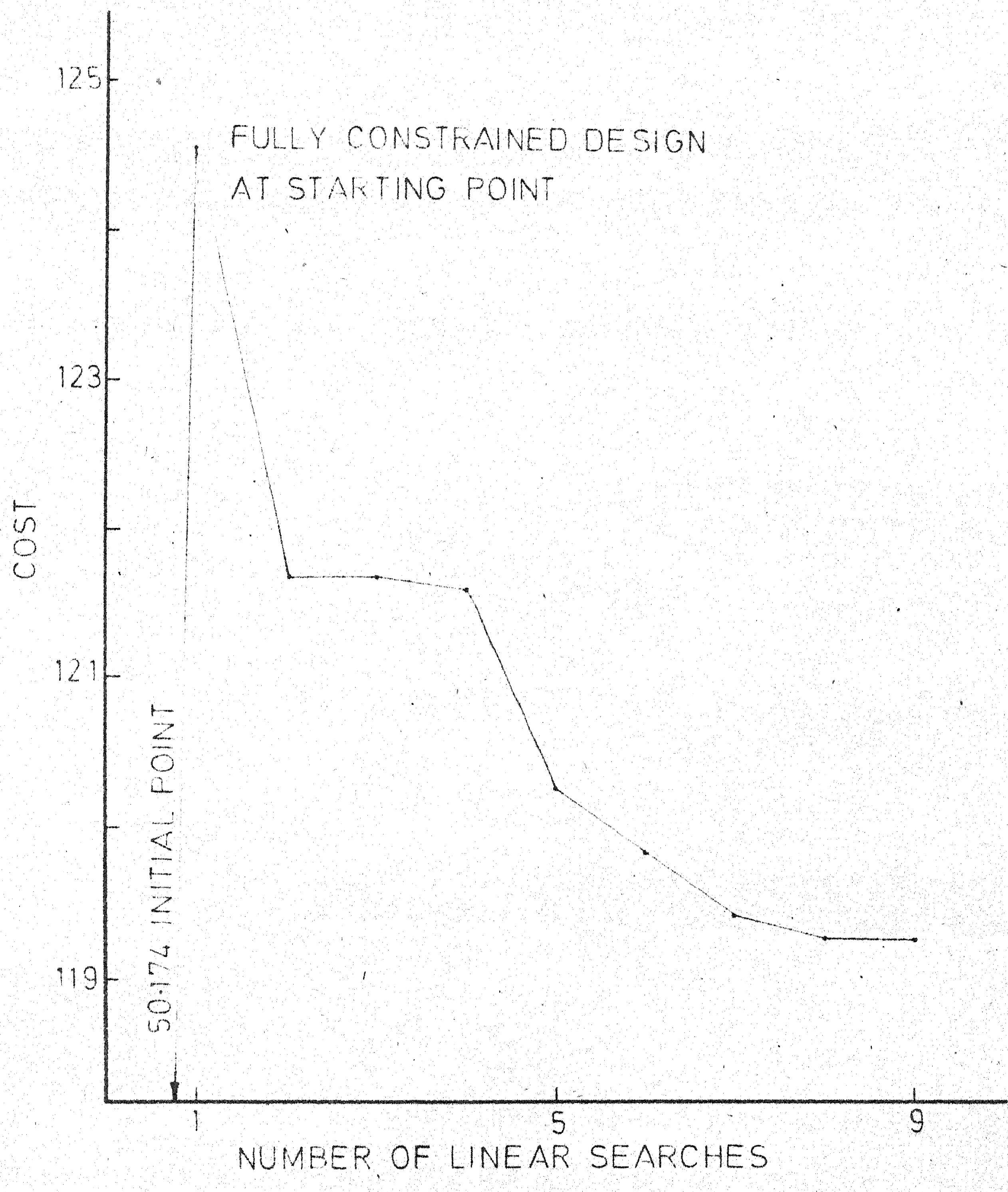


FIG.5.9 PROGRESS OF OPTIMIZATION-FULLY CONSTRAINED DESIGN APPROACH FOR PROBLEM 5

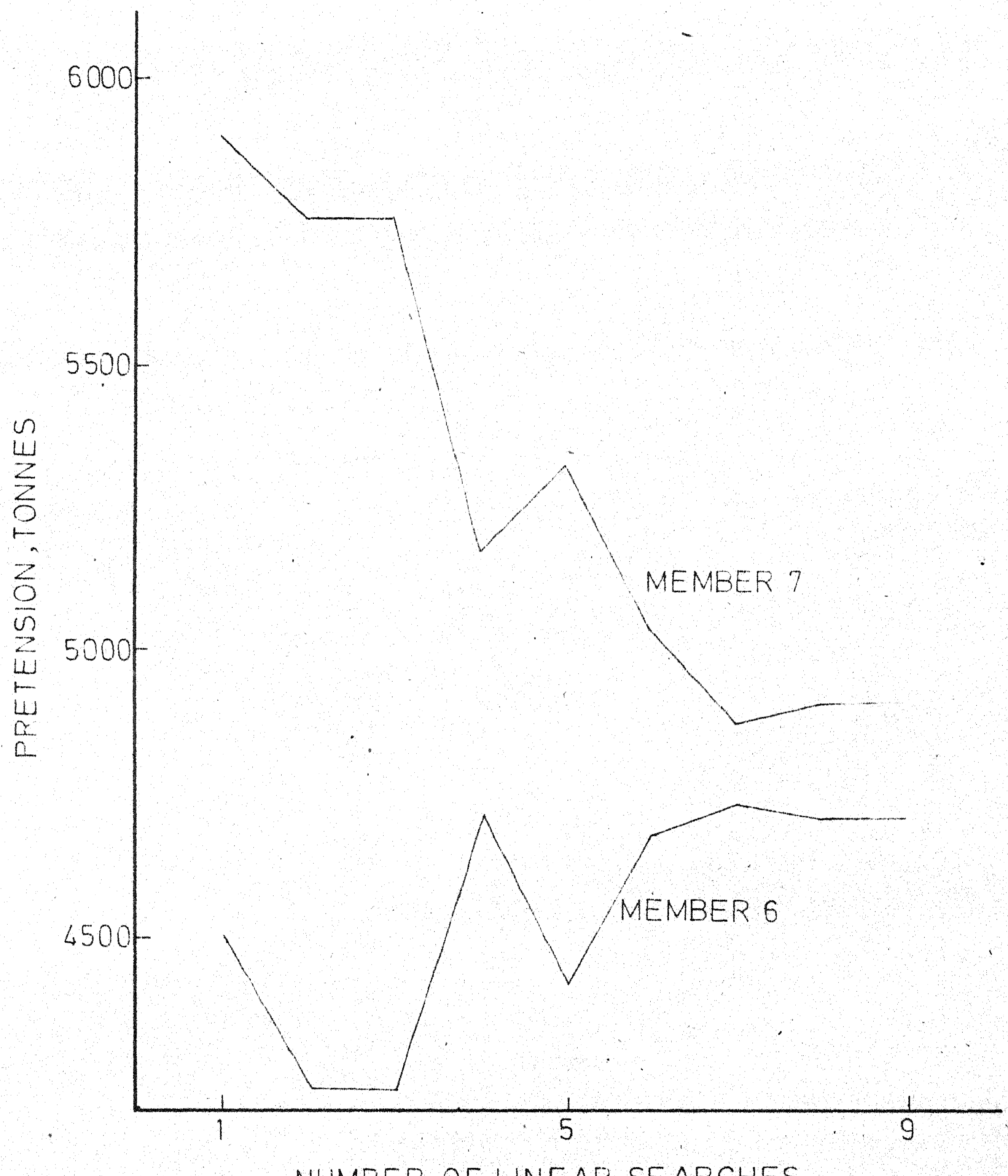


FIG. 5·10 PATH OF THE DESIGN VARIABLES-FULLY CONSTRAINED DESIGN APPROACH FOR PROBLEM 5

Table 5.13: Problem 6 - Effect of Variation of Depth of Girder - Linear Analysis.

Member	Qty	D = 150	D = 200	D = 250	D = 300	D = 350
1	t_w, cm	3.0833	2.3143	1.8585	1.5461	1.3273
	t_f	6.7112	4.9654	3.9044	3.1969	2.6922
2	t_w	2.6486	1.9886	1.5930	1.3230	1.2019
	t_f	8.9720	7.3020	6.3299	5.6792	5.1835
3	t_w	1.8563	1.4672	1.2107	1.2015	1.2030
	t_f	8.4158	7.3959	6.4277	5.7106	5.1831
4	t_w	1.5838	1.2046	1.2097	1.2007	1.2010
	t_f	1.8180	1.3371	1.2017	1.2002	1.2004
5	t_w	2.2196	3.7747	2.4439	2.6109	2.0193
	t_f	3.5046	2.0514	2.2956	2.0132	2.1365
6	A, cm^2	472.57	520.20	519.13	519.16	520.91
	F, Tonnes	4239.1	4681.7	4671.0	4672.3	4687.3
7	A	460.91	459.29	496.45	523.27	543.36
	F	4146.9	4130.8	4460.4	4708.2	4888.3
Cost		189.339	158.207	140.467	129.094	121.264
Volume of Beam (m^3)		174.865	142.829	124.681	112.998	104.903
Volume of Cable (m^3)		4.825	5.126	5.262	5.365	5.454

D is in cm.

Table 5.14: Problem 6 – Effect of Variation of Depth of Girder – Nonlinear Analysis.

Member	Qty.	D = 150	D = 200	D = 250	D = 300	D = 350
1	t_w, cm	3.0920	2.3207	1.9320	1.5594	1.3305
	t_f	6.6668	4.9407	3.3849	3.1914	2.6846
2	t_w	2.6550	1.9908	1.6730	1.3317	1.2053
	t_f	9.0810	7.3634	6.3314	5.6887	5.2013
3	t_w	1.8874	1.4402	1.7530	1.9153	1.2127
	t_f	8.2920	7.2760	6.3176	5.4684	5.1259
4	t_w	1.5879	1.2073	1.3792	1.2061	1.2053
	t_f	1.8116	1.3364	1.2028	1.2004	1.2021
5	t_w	4.1632	3.6720	3.8617	3.2901	2.5753
	t_f	3.3668	2.3447	1.7104	1.7365	1.9301
6	A, cm^2	457.27	505.99	521.98	517.80	516.51
	P, Tonnes	4114.5	4552.7	4694.4	4660.0	4647.8
7	A	481.01	475.71	492.76	524.52	551.08
	P	4326.8	4275.0	4434.5	4720.6	4653.1
Cost		190.332	158.692	141.502	129.792	121.537
Volume of Beam (m3)		175.923	143.399	125.704	113.709	105.173
Volume of Cable (m3)		4.803	5.097	5.266	5.361	5.455

D is in cm.

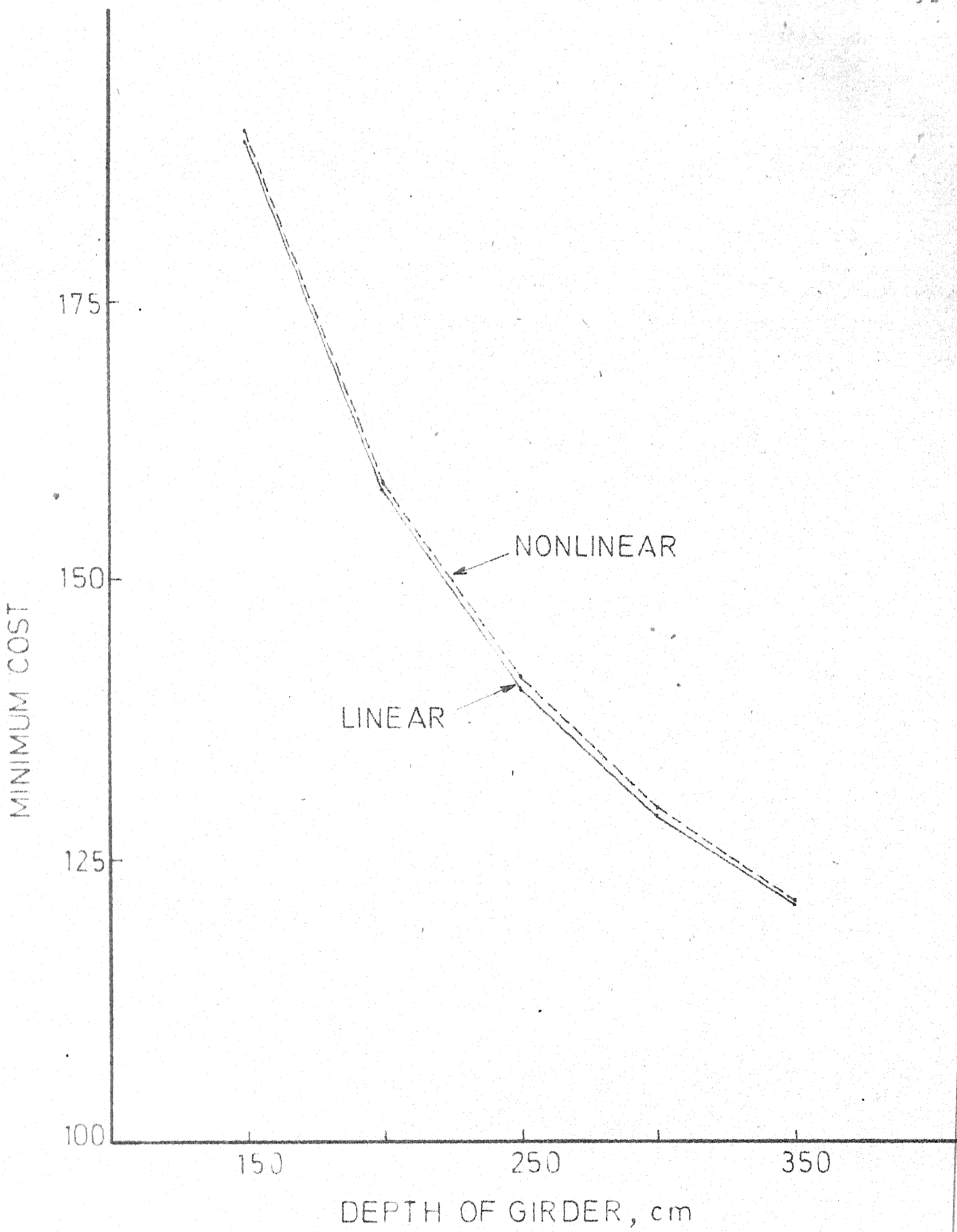


FIG.5.11 VARIATION OF MINIMUM COST WITH RESPECT TO DEPTH OF GIRDER-PROBLEM 6

Table 5.15: Problem 6 – Results of Lower Cable Stresses

D = 150 cm and σ_{yc} = 10000 kg/cm^2

Member	Qty.	Linear Analysis	Nonlinear Analysis
1	t_w, cm	3.0878	3.0896
	t_f	6.6962	6.6685
2	t_w	2.6489	2.6513
	t_f	9.0141	9.0772
3	t_w	1.8017	1.8718
	t_f	8.0130	8.3410
4	t_w	1.5871	1.5886
	t_f	1.8329	1.8101
5	t_w	1.2704	8.7356
	t_f	4.2073	2.3993
6	A, cm^2	648.18	651.39
	P, Tonnes	3888.5	3908.2
7	A	687.73	676.84
	P	4124.2	4060.1
Cost		195.458	196.990
Volume of Beam (m^3)		174.966	176.561
Volume of Cable (m^3)		6.831	6.810

CHAPTER SIX

DISCUSSIONS AND CONCLUSIONS

The results of the example problems presented in the previous chapter are discussed herein. The outcome of the numerical studies is described and conclusions drawn therefrom are enumerated. Finally, the suggestions to improve upon the techniques used in the present work and to make the formulations still more practically oriented are indicated.

6.1 COMPARISON OF THE FORMULATIONS:

The computational experience shows that the synthesis approach to the optimum design of the class of cable stayed structures studied is quite efficient and reliable. The fully constrained design formulation, as proposed in the present work, is certainly more efficient in the sense that it consumes only a fraction of the computer time taken by the synthesis approach to arrive at the optimum design. This is due to the fact that only one unconstrained minimization is needed in the case of the fully constrained design formulation. Furthermore, the number of design variables in this formulation is relatively small. This fact illustrates the point that the mathematical programming methods coupled with an optimality criterion based approach would be much faster than the simple use of mathematical programming techniques where an optimality criterion can be obviously identified.

The integrated synthesis approach has more number of design variables and constraints than the synthesis approach. This fact itself cancels out the advantages gained in not solving a set of simultaneous equations at every design point of the search procedure and the use of the much faster gradient computation procedure. The computational time is therefore always more than that for the synthesis approach.

The integrated synthesis approach successfully led to the optimal design for problems 1 and 2 but resulted in a premature termination of the program for one case in Problem 2 and for Problems 3, 4 and 5. This is due to the fact that during the first two or three unconstrained minimizations, the unbalanced forces and moments in the equilibrium equations, which actually form the equality constraint values are brought down from large values to almost zero. This is brought about without much regard to the objective function, which is the cost of the structure. It is found that once the equilibrium constraints, which are nonlinear equality constraints, are satisfied to such a degree, the path of optimization is along the curved intersection of the equality constraint surfaces. Tracing this curved path by straight move vectors leads to the premature termination of the algorithm. Such a strict satisfaction of the equilibrium constraints at the early stages of the optimization procedure is due to the fact that the ~~con~~

about an order of magnitude less than the starting values. This scheme turned out to be satisfactory for problems 1 and 2 only.

6.2 RESULTS OF THE LINEAR ANALYSIS:

The results based on linear analysis show that the optimum design for the example problems always results in a fully constrained design. This is characterised by the fact that,

a) the webs of the beams are always constrained by the maximum thickness limitation or by the shear stress limitation;

b) the flanges of the beams are always constrained by the maximum extreme fibre stress in combined bending and thrust or by the minimum thickness limitation, and

c) the areas of the cables are always constrained by the pretension stress or the final stress under working load.

The prestressing forces in the cables adjust themselves in such a way that the cost of the resulting design is minimum. This characteristic behaviour of this class of cable stayed structures, in fact, led to the fully constrained design formulation used in the present work.

Since the optimum design is a fully constrained design, it is natural to expect that the variations in the relative cos

of mild steel and high tensile cable should not have an influence on the total quantities of mild steel and cable. The results of Table 5.8 show that this is true. The changes in the quantities of mild steel and high tensile cable are only marginal and can be said to be negligible keeping the large variation in the relative costs from 1:3 to 1:5 in mind.

In case of structures with only nodal loads, the pre-tension forces adjust themselves in such a way that at the optimum design, the beam elements undergo very little bending and are mainly axially loaded. This is consistent with what could be intuitively expected. However, for structures with member loads no such conclusion is valid. The comparison of the optimum designs for the joint loaded and the member loaded cable stayed structures show an absolute lack of correlation, and the member loaded structure has a much higher cost. It is quite clear that lumping the loads at the nodes is not desirable, even in the analysis problems, and for the purposes of optimum design, the results of such lumping would lead to an unsafe design.

The tower member, at the optimum, is subjected only to an axial force and the bending is negligible. Therefore, only the area of cross section of the tower is of importance and its moment of inertia is only of secondary importance. The optimum design therefore does not show the typical characterist

of web thickness being at the minimum and the flange thickness being determined by the stress constraint. Any combination of the web and flange thicknesses such that the area of cross section is the required one produces a near-optimum design. In the synthesis approach, convergence may take place to any point in this plateau. This is the reason for the cross section of the beam in Problem 1 case b, and the cross section of the tower in Problems 3 to 6 being different in different methods. The area of cross section, however, may be observed to be almost the same in all the methods for each of the problems.

From the parametric study presented in Table 5.13 it is observed that the total cost of materials increases as the depth of the girder decreases. This increase is mainly due to the increase in the quantity of mild steel used. The quantity of high tensile cable used is seen to decrease, although only slightly, with the decrease in depth. The pretension forces do not show a specific trend, and their variation is rather small for the large variation in the depth. It is seen from experience with the fully constrained design formulation that the pretension forces, in the neighbourhood of the optimum design, do not significantly affect the cost and that there is a rather shallow region around the optimum. Therefore, a fully constrained design with pretension forces in a small range about the optimum would produce a near-optimum design.

It is for this reason that no specific trend would be observed in the values of the pretension forces.

Comparison of results presented in Tables 5.13 and 5.15 show that a reduction of the allowable stresses in the cable by $33\frac{1}{3}$ percent does not significantly affect the total quantity of mild steel used. The quantity of cable used increases nearly proportionally, and there is a slight readjustment of the pretension forces in the cables.

6.3 RESULTS OF THE NONLINEAR ANALYSIS:

Comparison of the results of the example problems using the linear and the nonlinear analysis procedures indicates only a marginal difference between the costs of the two optimum designs. However, it is consistently observed that considering the nonlinear effects in a joint loaded structure results in a reduction in the minimum cost, while in a structure with member loads an increase in the minimum cost is experienced. This is explained as follows:

In a joint loaded structure, the pretension forces at the optimum design are determined such that the structure is essentially axially loaded and the bending effects are small. When the beam-column effect is considered, the flexural stiffnes of the members is reduced, resulting in smaller design moments. The resulting design is therefore less costly. In a structure with member loads, the negative moments of the members govern

the design. The negative moments are mainly contributed by the fixed end moments due to the member loads, and the inclusio of beam-column effect increases the fixed end moments. Thus the design moments are larger, resulting in a costlier structure.

The influence of the nonlinear analysis of the structure on the optimum design is to alter very slightly the thicknesses of the flanges and the webs of the beam elements of the corresponding optimum design using the linear analysis. The nonlinearity effects, it seems, are counter balanced by a small variation in the pretension forces in the different cables in order to keep the cost to a minimum. Thus it is seen that for a smaller depth of the structure when the nonlinearity is more, the variation in the pretension forces between the linear and nonlinear analyses is also more. As the depth increases, the effect of nonlinearity decreases, and so does the change in the pretension forces. However, there is no specific trend in the results obtained.

6.4 FURTHER WORK:

An effort at the extension of the current work should be predominantly more practically oriented. The formulation for such a case should have at least two loading conditions, a. the dead load and the pretension in the cables, and b. the above loads plus live load. Depending on the live load chosen,

various live load positions may be necessary. The live load positions to give the worst live load effects may be found using the influence line diagrams. It may be worthwhile to specify the thickness of the web of box beams and to consider only the flange thickness as the design variable. The topological parameters, especially the height of the tower and the points of support of the girders by the cables could also be taken as the design variables.

These additions would certainly make the formulation more complex and the optimization procedure would be quite time consuming. The author feels that the Behaviour Model Method |76| as discussed in Section 4.3 would be an efficient approximation procedure to try and append to the synthesis approach. Since multiple loading conditions are involved, a nonlinear analysis for each loading condition would be too time consuming. Alternately, a certain reduction in the allowable stresses may be made to take care of the nonlinearity. It is, however, felt that since the optimum design arrived at through this effort would only be a skeletal structure, considering only the linear elastic behaviour would be justified.

An interesting point to observe from such studies is, whether an optimality criterion like the fully constrained design emerges out from these extensions also. In such a case, a formulation combining the resulting optimality criterion and

the optimization procedure with a reduced number of variables would be very efficient. Such a procedure can also incorporate, with little effort, the availability of discrete thicknesses of plates, and the discrete variation of thickness of the flange within the length of a member.

A study of the effect of the topological configuration on the cost of the structure would be very meaningful. The cables can be arranged in fan shape, harp shape or any other arbitrary shape. Study of the relative economy of these, similar to the one made by Leonhardt and Zellner |20|, would be very instructive. It would also be interesting to find the effect of the number of cables used on the optimum cost of the structure. Intuitively, it is felt that the increase in the number of cables would decrease the total cost of materials used.

The synthesis approach and the program developed can be generalised and extended to the optimum design of guyed towers. This would require the grouping of the members having the same design variables and modification of the program to handle such a grouping. The three dimensional stiffness analysis procedure would have to be incorporated in the program. The effect of large deformation should be included in the non-linear analysis procedure for this class of structures. The nonlinearities in the behaviour of guyed towers, both due to large deformation and due to beam-column effect are reported to be significant.

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The following abbreviations have been used in the names of the journals.

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JEMD, ASCE	-	Journal of the Engineering Mechanics Division, Proceedings, ASCE
IJNME	-	International Journal for Numerical Methods in Engineering
Proc. ICE	-	Proceedings, Institution of Civil Engineers, London.

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APPENDIX A

SECANT STIFFNESS MATRIX AND ITS DERIVATIVES

A.1 The Stiffness Matrix S for a bending element in the element coordinate system is given as:

$$S \equiv \begin{bmatrix} \frac{AE}{L} & 0 & 0 & -\frac{AE}{L} & 0 & 0 \\ & \frac{2EI(C_1+C_2)}{L^3} & \frac{EI(C_1+C_2)}{L^2} & 0 & -\frac{2EI(C_1+C_2)}{L^3} & \frac{EI(C_1+C_2)}{L^2} \\ & & \frac{EI}{L}(C_1) & 0 & -\frac{EI(C_1+C_2)}{L^2} & \frac{EI}{L}\,C_2 \\ & & & \frac{AE}{L} & 0 & 0 \\ & \text{SYMMETRIC} & & & \frac{2EI(C_1+C_2)}{L^3} & -\frac{EI(C_1+C_2)}{L^2} \\ & & & & & \frac{EI}{L}\,C_1 \end{bmatrix} \qquad (A.1)$$

where C_1 and C_2 for various cases are as follow:

a) For zero axial force or when axial force is ignored which leads to linear analysis

$$C_1 = 4 \quad ; \quad C_2 = 2 \qquad (A.2)$$

b) For compressive axial force,

$$C_1 = \frac{\phi(\sin\phi - \phi\cos\phi)}{2(1-\cos\phi) - \phi\sin\phi} \qquad (A.3)$$

$$C_2 = \frac{\phi(\phi - \sin\phi)}{2(1-\cos\phi) - \phi\sin\phi} \qquad (A.4)$$

in which $\phi^2 = \frac{QL^2}{EI}$ where Q is the axial force in compression.

c) For tensile axial force,

$$C_1 = \frac{\psi(\psi\cosh\psi - \sinh\psi)}{2(1-\cosh\psi) + \psi\sinh\psi} \qquad (A.5)$$

$$C_2 = \frac{\psi(\sinh\psi - \psi)}{2(1-\cosh\psi) + \psi\sinh\psi} \qquad (A.6)$$

in which $\psi^2 = \frac{QL^2}{EI}$ where Q is the axial force in tension.

For the purpose of this work only cases a and b are required. The fixed end moment under uniform load for an element in axial compression is given by,

$$\text{fixed end moment} = \pm \frac{qL^2}{12}\left[\frac{\tan u - u}{\frac{1}{3}u^2\tan u}\right] \qquad (A.7)$$

where, q is the u.d.l. per unit length, and $u = \frac{\phi}{2}$.

A.2 Computation of derivatives of the stiffness matrix with respect to the design variables:

Three typical expressions are involved in the elements of the stiffness matrix. The derivatives of these expressions are

computed, and the derivative of the stiffness matrix computed therefrom.

Let $E_1 = \frac{EA}{L}$. Its derivative with respect to a variable x_i is,

$$\frac{\partial E_1}{\partial x_i} = \frac{E}{L} \cdot \frac{\partial A}{\partial x_i} \tag{A.8}$$

$$\text{Let,} \quad E_2 = \frac{EI}{L} C_1 \tag{A.9}$$

$$\frac{\partial E_2}{\partial x_i} = \frac{\partial E_2}{\partial I} \cdot \frac{\partial I}{\partial x_i} = \left(\frac{E}{L} C_1 + \frac{EI}{L} \cdot \frac{\partial C_1}{\partial I} \right) \frac{\partial I}{\partial x_i} \tag{A.10}$$

$$\frac{\partial C_1}{\partial I} = \frac{\partial C_1}{\partial \phi} \cdot \frac{\partial \phi}{\partial I} \tag{A.11}$$

$$\frac{\partial E_2}{\partial x_i} = \left(\frac{E}{L} C_1 + \frac{EI}{L} \cdot \frac{\partial C_1}{\partial \phi} \cdot \frac{\partial \phi}{\partial I} \right) \frac{\partial I}{\partial x_i} \tag{A.12}$$

Now,

$$C_1 = \frac{\phi(\text{Sin } \phi - \phi \text{ Cos } \phi)}{2(1-\text{Cos } \phi) - \phi \text{ Sin } \phi} = \frac{N1}{D1} \tag{A.13}$$

$$\frac{\partial C_1}{\partial \phi} = \frac{D1\,(\text{Sin } \phi - \phi \text{ Cos } \phi + \phi^2 \text{ Sin } \phi) - N1\,(\text{Sin } \phi - \phi \text{ Cos } \phi)}{D1^2} \tag{A.14}$$

$$\text{Now,} \quad \phi^2 = \frac{QL^2}{EI} \quad \text{or} \quad \phi = \sqrt{\frac{QL^2}{EI}} \tag{A.15}$$

Assuming that the axial force Q does not change due to a change in I,

$$\frac{\partial \phi}{\partial I} = - \sqrt{\frac{QL^2}{E}} \cdot \frac{1}{2I^{3/2}} = - \frac{\phi}{2I} \tag{A.16}$$

$\therefore$ $\frac{\partial E_2}{\partial I}$ is obtained by substituting the quantities from A.16 and A.14 in A.12.

Similarly, let,

$$E_3 = \frac{EI}{L} C_2 \tag{A.17}$$

$$\frac{\partial E_3}{\partial x_i} = \left(\frac{E}{L} C_2 + \frac{EI}{L} \cdot \frac{\partial C_2}{\partial \phi} \cdot \frac{\partial \phi}{\partial I} \right) \frac{\partial I}{\partial x_i} \tag{A.18}$$

$$C_2 = \frac{\phi(\phi - \text{Sin } \phi)}{2(1-\text{Cos } \phi) - \phi \text{ Sin } \phi} = \frac{N2}{D1} \tag{A.19}$$

$$\frac{\partial C_2}{\partial \phi} = \frac{D1(2\phi - \phi \text{ Cos } \phi - \text{Sin } \phi) - N2 \text{ (Sin } \phi - \phi \text{ Cos } \phi)}{D1^2} \tag{A.20}$$

$\frac{\partial E_3}{\partial x_i}$ is obtained by substituting the quantities from A.20 and A.16 in A.18.

$\frac{\partial I}{\partial x_i}$ and $\frac{\partial A}{\partial x_i}$ are calculated knowing the type of cross section and the design variable.